Introduction to Rational Functions

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Rational Numbers and Polynomials

Recall what a rational number is. A **rational number** is a number that can be written as a fractions. e.g.

$$\frac{3}{5}, \frac{1}{4}, -\frac{7}{10}, \frac{2}{3}$$

Now, let's recall what a **polynomial** is. A polynomial is an expression of more than two algebraic terms. For example,

$$3 + x, 2 - x + 4x^2, x^2 - 3x^5 + 6$$

Rational Expression or Function

A **rational expression or function** is an ''albegraic fraction''. That is, a fraction of polynomials, where the numerators and denominator are polynomials. e.g.

$$\frac{1}{x}, \frac{y^2 - 1}{y^2 + 2y + 1}, \frac{a}{a + 2}, \frac{x^2 - 4}{1} = x^2 - 4$$

Example

Which of the following expressions are NOT rational expressions/functions?

(a) $\frac{\sin x}{\cos x + 4}$

(b) $\sqrt{4x^2 - 1}$

(c) $9 + x^2 + x^4$

(d) $\frac{1}{\sqrt{x}}$

Solution

(a), (b) and (d) are NOT rational expressions. Why?

For (a), $\sin x$ and $\cos x$ are not polynomials.

For (b) $\sqrt{4^2 - 1}$ is not a polynomial.

For (d), $\sqrt{x}$ is not a plynomial.

Non-permissible Values or Restrictions for a Rational Function

Now, let's consider our rational numbers again. Recall when a fraction or rational number is NOT defined. A rational number is NOT defined when the denominator is zero. Keepign this in mind, let's see when the following rational expressions would NOT be defined.

Example

When are the following rational expressions NOT defined?

(a)

$$\frac{x-3}{x^2-2x+1}$$

(b)

$$\frac{1-t}{t^2-1}$$

Solution

(a) We want to determine when the rational expressions $\frac{x-3}{x^2-2x+1}$ is NOT defined. Recall, that any fraction or rational number is NOT defined when the denominator is equal to zero This is because, .we cannot divide by zero. With this in mind, when does our rational expression have a denominator equal to zero?

The denominator for this raitional expression is $x^2 - 2x + 1$. Let's figure out when it is equal to zero. That is, what values of x make $x^2 - 2x + 1$ equal to zero.

$$\begin{aligned} x^2 - 2x + 1 &= 0 \\ (x-1)(x-1) &= 0 \end{aligned}$$

Therefore, when $x = 1$, then $x^2 - 2x + 1 = 0$. This means the rational expressions $\frac{x-3}{x^2-2x+1}$ is NOT defined when $x = 1$. The value $x = 1$ is a *non-permissible value* or a *restriction* for the rational expression $\frac{x-3}{x^2-2x+1}$.

(b) We want to find the non-permissible values or restrictions for the rational function $\frac{1-t}{t^2-1}$. Let's figure out when the denominator, $t^2 - 1 = 0$.

$$\begin{aligned} t^2 - 1 &= 0 \\ (t-1)(t+1) &= 0 \end{aligned}$$

Therefore, $t = 1, t = -1$ are the restrictions on the rational function $\frac{1-t}{t^2-1}$. Note the following,

$$\begin{aligned} &\frac{1-t}{t^2-1} \\ = &\frac{-(t-1)}{(t-1)(t+1)} \\ = &\frac{-(t-1)}{(t-1)(t+1)} \\ = &-\frac{1}{t+1} \end{aligned}$$

Therefore, $t = -1$ and $t = 1$ are the non-premissiable values or restrictions for $\frac{1-t}{t^2-1}$

Exercises

1. Which of the following are rational expressions/functions?
   (a) $\frac{3+x}{x^2-1}$
   (b) $\frac{\sin x}{\tan x}$
   (c) $\sqrt{\frac{x^2+4x+4}{9+x^2}}$
   (d) $\frac{e^x}{4x+9}$
2. For those that are NOT rational functions in #1, explain why in each case,
3. Determine the restrictions for each of the following rational functions.
   (a) $\frac{1}{x^2-1}$
   (b) $\frac{2a}{3+a}$
   (c) $\frac{1-x}{(x-)(x-1)}$
   (d) $\frac{k^2-1}{k^2+2k+2}$
   (e) $\frac{6x-8}{(3x-4)(2x+5)}$
4. Simplify the following rational expressions and state any restrictions.
   (a) $\frac{x^2-49}{(2x-1)(x-7)}$
   (b) $\frac{5(a^2-a-6)}{10(3-a)(a+2)}$
   (c) $\frac{6r^2p^2}{4rp^4}$
   (d) $\frac{10k^2+55k+75}{20k^2-10k-150}$
   (e) $\frac{(x+2)^2-(x+2)-20}{x^2-9}$
   (f) $\frac{(x^2-x)^2-8)x^2-x)+12}{(x^2-4)^2-(x-2)^2}$