Rational Equations

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Rational Equations

Let's recall what an eqution is, as we know it to be.

$$\begin{aligned} x^2+3x-6 &= 0 \\ (x-2)(x^2+1) &= 3x-1 \\ y^2+1 &= 2y-4 \\ x^3+4x^2 &= 2x+7 \end{aligned}$$

All the above are examples of "equations". An **equation** has an equal sign "=" in it and a polynomial on either side of the equals sign. A **rational equation** is similar except now there is at least one rational function on either side of the equals sign. Note, a polynomial is a rational function with a denominator of 1. So we want to consider rational functions with a denominator different from 1. Some examples of rational equtions are given below.

$$\begin{aligned} x &= \frac{x-3}{x+1} \\ \frac{x}{4}-\frac{7}{x} &= 3 \\ \frac{2}{z^2-4}-\frac{10}{6z+12} &= \frac{1}{z-2} \end{aligned}$$

Now, let's *solve* a rational equation.

Example

Solve the following rational equation:

$$\begin{aligned} x &= \frac{x+3}{x+1} \\ x(x+1) &= x+3, \quad \text{cross multiply by } x+1 \\ x^2+x &= x+3 \\ x^2+x-x-3 &= 0 \\ x^2-3 &= 0 \\ (x+\sqrt{3})(x-\sqrt{3}) &= 0 \\ \therefore x &= \pm\sqrt{3}, \ x \neq -1 \end{aligned}$$

where restrictions are given by $x \neq -1$.

Example

Solve the following rational equation, stating all restrictions.

$$\begin{aligned}
\frac{e-y}{3y}+\frac{1}{4} &= \frac{1}{2y} \\
\frac{(3-y)(4)}{(3y)(4)}+\frac{3y}{4(3y)} &= \frac{1}{2y} \\
\frac{12-4y+3y}{12y} &= \frac{1}{2y} \\
\frac{12-y}{12y} &= \frac{1}{2y}, \quad \text{cross multiply by } 2y \\
\frac{(12-y)(12y)}{12y} &= 1 \\
\frac{12-y}{6} &= 1 \\
12-y &= 6 \\
12-6-y &= 0 \\
6-y &= 0 \\
6 &= y,\ y \neq 0
\end{aligned}$$

Exercises

1. Solve the following rational equations. State all restrictions.

   (a)
   $$\frac{2x+3}{x+5}+\frac{1}{2}=\frac{7}{2x+10}$$

   (b)
   $$\frac{f+3}{2}-\frac{f-2}{3}=2$$

   (c)
   $$\frac{d}{d+4}=\frac{2-d}{d^2+3d-4}+\frac{1}{d-1}$$

   (d)
   $$\frac{-3y}{y-1}+6=\frac{6y-9}{y-1}$$

   (e)
   $$\frac{2b}{b+5}=1+\frac{3}{b+2}$$

   (f)
   $$\frac{2}{x-1}-3=\frac{5x}{x+1}$$

   (g)
   $$\frac{5}{x}-\frac{1}{x-1}=\frac{1}{x-1}$$

2. The sum of the reciprocals of two consecutive integeters is $\frac{11}{30}$. What are the integers?

3. Two consecutive numbers are given by x and $x+1$. If 6 is added to the first and 2 subtracted from the second, and the quotient of the new nubmers is $\frac{9}{2}$, determine the numbers algebraically.