

Multiplying and Dividing Rational Functions

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Example: Multiplying and Dividing Rational Numbers

Let's start by reviewing multiplication and division of rational numbers. For example,

1.

$$\begin{aligned} & -\frac{3}{2} \times \frac{5}{13} \\ &= \frac{(-3)(5)}{(2)(13)} \\ &= -\frac{15}{26} \end{aligned}$$

2.

$$\begin{aligned} & \left(\frac{4}{5}\right) \left(\frac{3}{2}\right) \\ &= \frac{(4)(3)}{(5)(2)} \\ &= \frac{12}{10} = \frac{6}{5} \end{aligned}$$

In the first example we multiplied the numerators together and the denominators together. In the second example we simplified, looking for common factors in the numerator and denominator, cancelling them out after multiplying numerators and denominators together.

Now let's consider multiplying rational functions.

Example: Multiplying Rational Functions

Multiply and simplify the following rational expressions.

(a) $\frac{4x^2}{3xy} \times \frac{y^2}{8x}$

(b) $\frac{y^2-9}{r^3-r} \times \frac{r^2-r}{y+3}$

Solution

(a)

$$\begin{aligned} & \left(\frac{4x^2}{3xy}\right) \left(\frac{y^2}{8x}\right) \\ &= \frac{4x^2y^2}{24x^2y} \\ &= \frac{y}{6}, \end{aligned}$$

and the restrictions are $x \neq 0$, $y \neq 0$.

(b)

$$\begin{aligned} & \left(\frac{y^2 - 9}{r^3 - r}\right) \left(\frac{r - r}{y + 3}\right) \\ &= \frac{(y - 3)(y + 3)r(r - 1)}{r(r^2 - 1)(y + 3)} \\ &= \frac{(y - 3)}{(r + 1)} \\ &= \frac{y - 3}{r + 1}, \end{aligned}$$

and the restrictions are $r \neq 0, 1, -1$, $y \neq -3$ because this is when the denominator equals zero.

Now let's review the division of rational numbers. Let's consider an example.

$$\frac{14}{3} \div \frac{7}{6} = \frac{14}{3} \times \frac{6}{7} = \frac{2}{1} \times \frac{2}{1} = 4$$

When dividing one fraction or rational number by the other, notice that the dividend is multiplied by the reciprocal of the divisor. Now the exercise is multiplication of fractions, which we know how to do. Now let's try dividing rational functions.

Examples of Dividing Rational Functions

$$\begin{aligned} & \frac{3x^2}{y^2} \div \frac{x}{y} \\ &= \frac{3x^2}{y^2} \times \frac{y}{x} \\ &= \frac{3x^2y}{y^2x} \\ &= \frac{3x}{y}, \end{aligned}$$

where $x \neq 0$ and $y \neq 0$. Let's look at another example,

$$\begin{aligned} & \frac{x^2 - 4}{x^2 - 4x} \div \frac{x^2 + x - 6}{x^2 + x - 20} \\ &= \frac{x^2 - 4}{x^2 - 4x} \times \frac{x^2 + x - 20}{x^2 + x - 6} \\ &= \frac{(x - 2)(x + 2)(x + 5)(x - 4)}{x(x - 4)(x + 3)(x - 2)} \\ &= \frac{(x + 2)(x + 5)}{x(x + 3)}, \quad x \neq 0, -5, 4, -3, 2 \end{aligned}$$

Exercises

1. Simplify the following and identify all restrictions.

(a)

$$\frac{12mf}{5cf} \times \frac{15c}{4m}$$

(b)

$$\frac{(y-7)(y-3)}{(2y-3)(2y+3)} \times \frac{4(2y+3)}{(y+3)(y-1)}$$

(c)

$$\frac{a+3}{a+1} \div \frac{a^2-9}{a^2-1}$$

(d)

$$\frac{r^2-7r}{r^2-49} \div \frac{3r^2}{r+7}$$

(e)

$$\frac{2x-6}{x+3} \times \frac{x+3}{2}$$

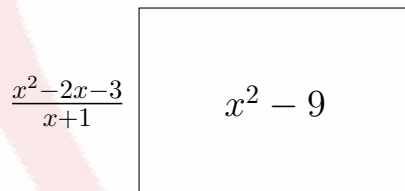
(f)

$$\frac{v-5}{v} \div \frac{v^2-2v-15}{v^3}$$

(g)

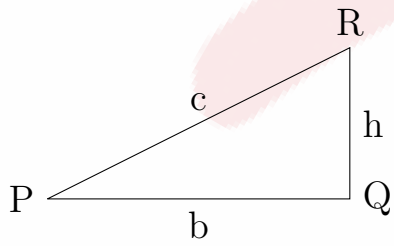
$$\frac{9x^2-1}{x+5} \div \frac{3x^2-5x-2}{2-x}$$

2. Write an expression to represent the length of the rectangle where the area, A , and width, w , are given below, Simplify your answer and state any restrictions.



$$A = x^2 - 9,$$
$$w = \frac{x^2 - 2x - 3}{x + 1}$$

3. Give an expression for the area of a $\triangle PQR$ where the base and height are given in the diagram below.



where,

$$h = \frac{x+2}{x-8}, \quad b = \frac{x^2-7x-8}{x^2-4}$$