

Adding and Subtracting Rational Functions

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July 4, 2024

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Let's start by reviewing the addition and subtraction of rational numbers. We'll start with an example.

Example

$$\frac{4}{2} + \frac{3}{7}$$

We need to start by finding a common denominator for the two fractions $\frac{4}{2}$ and $\frac{3}{7}$.

$$\begin{aligned} & \frac{4}{2} + \frac{3}{7} \\ &= \frac{4(7)}{2(7)} + \frac{3(2)}{7(2)} \\ &= \frac{28}{14} + \frac{6}{14} \end{aligned}$$

Once we have a common denominator which in this case is $2 \times 7 = 14$, we can add the numerators.

$$\begin{aligned} &= \frac{28 + 6}{14} \\ &= \frac{34}{14} \\ &= \frac{17}{7} \end{aligned}$$

After simplifying, factoring out common factors from the numerator and denominator, we have our final answer, $\frac{17}{7}$.

Example

Let's consider another example. This time subtraction of two fractions.

$$\begin{aligned} & \frac{3}{8} - \frac{2}{3} \\ = & \frac{3(3)}{8(3)} - \frac{2(8)}{3(8)} \\ = & \frac{9}{24} - \frac{16}{24} \\ = & \frac{9 - 16}{24} \\ = & \frac{-7}{24} \end{aligned}$$

Now let's look at the addition and subtraction of rational functions.

Example

Add the following rational expressions.

(a)

$$\frac{x^2}{x-2} + \frac{3x}{x-2} - \frac{10}{x-2}$$

(b)

$$\frac{2x}{xy} + \frac{4}{x^2}$$

Solution

(a) The denominators are all the same in this example so all we have to do is add and simplify the numerators.

$$\begin{aligned} & \frac{x^2}{x-2} + \frac{3x}{x-2} - \frac{10}{x-2} \\ = & \frac{x^2 + 3x - 10}{x-2} \\ = & \frac{(x+5)(x-2)}{x-2} \\ = & x+5, \end{aligned}$$

with restrictions $x \neq 2$.

- (b) In this second example, the denominators are different so we need to find a common denominator first.

$$\begin{aligned} & \frac{2x}{xy} + \frac{4}{x^2} \\ &= \frac{2x(x)}{(xy)(x)} + \frac{4(y^2)}{x^2y} \\ &= \frac{2x^2}{x^2y} + \frac{4y^2}{x^2y} \\ &= \frac{2x^2 + 4y^2}{x^2y}, \end{aligned}$$

with restrictions $x \neq 0, y \neq 0$.

Exercises

1. Add or subtract the following. Simplify and identify all restrictions.

(a) $\frac{7}{x} + \frac{3}{x}$

(b) $\frac{a^2}{a-4} - \frac{16}{a-4}$

(c) $\frac{x-5}{x^2+8x-20} - \frac{2x+1}{x^2-4}$

(d) $\frac{4}{9-x^2} - \frac{7}{3+x}$

(e) $\frac{6xy}{a^2b} - \frac{2x}{ab^2y} + 1$

(f) $\frac{2h}{h^2-a} + \frac{h}{h^2+6h+9} - \frac{3}{h-3}$

(g) $\frac{1}{x^2-x-12} + \frac{3}{x+3}$

(h) $\frac{ex+15}{x^2-25} + \frac{4x^2-1}{2x^2+9x-5}$

2. Simplify and state restrictions.

(a)

$$\frac{2x^2 - x}{x^2 - 3x} \times \frac{x^2 - x - 12}{2x^2 - 3x + 1} - \frac{x - 1}{x + 2}$$

(b)

$$\frac{\frac{3}{2} + \frac{3}{t}}{\frac{t}{t+6} - \frac{1}{t}}$$

(c)

$$\frac{\frac{1}{x+4} + \frac{1}{x-4}}{\frac{x}{x^2-16} + \frac{1}{x+4}}$$