Geometric Series

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Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$
$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$
$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i,\ i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$\begin{aligned} S_n &= t_0 + t_1 + \cdots + t_n \\ &= a + nar + ar^2 + \cdots + ar^n \\ &= a(1 + r + r^2 + \cdots + r^n) \\ &= a\left(\frac{1 - r^n}{1 - r}\right) \end{aligned}$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1-r^n}{1-r},\ r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \quad (1)$$
$$S_n = \frac{a - t_n}{1 - r} \quad (2)$$

Exercises

1. Find the following sum,
$$\sum_{n=1}^{7} 2^{n-1}$$

2. $$\frac{1}{2} + \frac{7}{4} + \frac{49}{8} + \cdots + \frac{2401}{32} = ?$$

3. Find the sum of the first 8 terms of,
$$3, 6, 12, \ldots$$

4. Find the sum of,
$$S_1 = 1 - \frac{1}{\sqrt{2}} + \frac{1}{2} - \frac{1}{2\sqrt{2}} + \frac{1}{4} - \frac{1}{4\sqrt{2}} + \frac{1}{8}$$

5. The finite geometric sequence has 10 terms. The sum of all the terms with even index is 682 and the sum of the all terms with odd index is 1364. Determine the first term and quotient, r, of the sequence.

6. The sum of a_1 and a_3 is 15. The sum of the first 3 terms of the sequence is 21. Determine the first term and the quotient, r.

