Geometric Series

RaiseMyMarks.com

2021

Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$
$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$
$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i,\ i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$\begin{aligned} S_n &= t_0 + t_1 + \cdots + t_n \\ &= a + nar + ar^2 + \cdots + ar^n \\ &= a(1 + r + r^2 + \cdots + r^n) \\ &= a\left(\frac{1 - r^n}{1 - r}\right) \end{aligned}$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1-r^n}{1-r},\ r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \quad (1)$$
$$S_n = \frac{a - t_n}{1 - r} \quad (2)$$

Exercises

1. Find the sum S_10 for the geometric series given the information below,

   a) $S_n = 2^n$

   b) $a_4 = 4,\ a_8 = 16$

   c) $a_2 = 4,\ a_3 = -8$

   d) $a_1 = -1,\ r = 0.3$

   e) $a_1 = 2,\ r = 3$

   f) $r = 2,\ S_7 = 508$

   g) $5a_2 - a_4 = 24,\ a_5 = 9_3$

   h) $a_2 = \frac{3}{2},\ a_7 = -\frac{3}{64}$