Instantaneous Rate of Change

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Suppse the following table represents a rocket's height in metres (m) over time in seconds (s).

Interval	Δh	Δt	$\frac{\Delta h}{\Delta t}$
$1 \leq t \leq 2$	10.3	1	10.3
$1 \leq t \leq 1.5$	6.375	0.5	12. 75
$1 \leq t \leq 1.1$	1.471	0.1	14.71
$1 \leq t \leq 1.01$	0.15151	0.01	15. 151
$1 \leq t \leq 1.001$	0.0151951	0.001	15.1951

Notice that as the interval gets smaller and closer to 1, the average rate of change, while increasing, approaches a fixed value, a limiting value, okf 15.2 m/s. This limiting value is called the *instantaneous rate of change.*

Instantaneous Rate of Change

For the function $y = f(x)$ the *instantaneous rate of change* of y with respect to x at point (x_1, y_1) is the limiting value of the average rates of change as the interval between the x-coordinate of point (x_1, y_1) and (x_2, y_2) continuously decreases to 0. The *instataneous rate of change* is given by,

$$\begin{aligned}\text{instantaneous rate of change} &= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \\ &= \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} \\ &= \lim_{x_1 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}\end{aligned}$$

Exercises

1. For $f(x) = x^2$, determine the average rate of change on the interval,
   (a) $1 \leq t \leq 6$
   (b) $1 \leq t \leq 2$
   (c) $1 \leq t \leq 1.5$
   (d) $-2 \leq t \leq 2$
   (e) $1 \leq t \leq 1.01$
   (f) $1 \leq t \leq 1.25$
   (g) $1 \leq t \leq 1.1$

   then estimate the instantaneous rate of change of $f(x)$ at $x = 1$ using the above results.

2. For the function $f(x) = 2x^2 - 1$ complete the table below.

Interval	$Delta f(x)$	Δx	$\frac{\Delta f(x)}{\Delta t}$
$1 \leq x \leq 2$			
$1.5 \leq x \leq 2$			
$1.75 \leq x \leq 2$			
$1.9 \leq x \leq 2$			
$1.95 \leq x \leq 2$			

   Then estimate the instantaneous rate of change when $x = 2$.

3. Determine the average rate of change of $g(x) = 4x^3 - 5x + 1$ on the intervals,
   (a) $3 \leq x \leq$
   (b) $4 \leq x \leq 5$
   (c) $4.5 \leq x \leq 5$
   (d) $4.75 \leq x \leq 5$

4. The volume of a cubic crystal grown in a laboratory is modelled by $V(x) = x^3$ where V is the volume in cubic centimetres and x is the side length. Estimate the instantaneous rate of chnage of volume when the side length is 5cm.

5. The volume of a cubic crystal grown in a laboratory is modelled by $V(x) = x^3$ where V is the volume in cubic centimetres and x is the side length. Find the average rate of change in the volume of the crystal w.r.t side length as each side grows from 4cm to 5cm.

6. From a platform tower 10m high, a diver performs a handstand dive. His height h in metres above the water at t seconds can be modelled by $h(t) = 10 - 4.9t^2$. Estimate the rate of change at which the diver enters the water.

7. A stone is dropped from a bridge that is 20cm above a river. The table on the left gives the height of the falling stone above the water's surface. An algebraic model for the data is the polynomial function $h(t) = -4.9t^2 + 20$, where h is the height above the water, in metres, and t is the elapsed time in seconds and $t \geq 0$. Estimate the instantaneous rate of change of height with respect to time at 1s.

8. A skydiver jumps from an airplane. Before she opens her parachute, she is in free fall. The function $d(t) = 4.9t^2$ models the vertical distance, d, in metres she has travelled at t seconds. What does the rate of change of distance with respect to time represent? What units are used to measure this rate of change? Estimate the rate of change at exactly 2 s.

9. Concentric cirlces from when a stone is dropped into a pool of water. How fast is the area changing with respect to radius when the radius is 120cm?

10. A crystal in the shape of a cube is gowing in a test tube. Estimate the rate at which the surface area is changing with respect to the side length when the side length of the crystal is 3cm.