

Equation of a Line (Part 2)

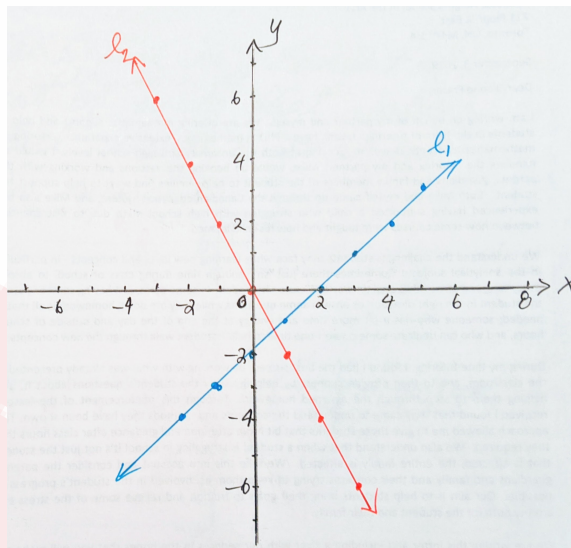


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Equation of a Line

A line can be thought of as the extension of a set of points that when joined create a straight line segment.



We have two lines l_1 and l_2 . What are the slopes of l_1 and l_2 , respectively? We need to find two points on l_1 and then calculate the *rise* and *run*. Two points on l_1 : $(x_0, y_0) = (1, -1)$ and $(x_1, y_1) = (4, 2)$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{2 - (-1)}{4 - 1} = \frac{3}{3} = 1$$

Therefore, the slope of line l_1 is 1. We usually represent the slope by m . So let's let the slope of l_1 be $m_1 = 1$.

Calculate slope of l_2 . Two points on l_2 : $(x_0, y_0) = (-1, 2)$ and $(x_1, y_1) = (2, -4)$.

$$\text{slope of } l_2 = m_2 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{-4 - 2}{2 - (-1)} = \frac{-6}{3} = -2$$

Therefore, the slope of line l_2 is $m_2 = -2$.

The *equation of a line* in general is given by,

$$y = mx + b$$

where m is the slope of the line and b is the y-intercept of the line.

What is the y-intercept?

The y-intercept is the point where the line crosses the y-axis. It can also be calculated by substituting $x = 0$ into the equation of a line. We can also find the y-intercept by plugging the coordinates of a point on the line into the equation for the line.

Let's try and find the y-intercept of l_1 . We have the slope for l_1 as $m_1 = 1$. The equation of line l_1 so far is,

$$y = m_1x + b_1 = x + b_1$$

where b_1 is the y-intercept of l_1 . Let's take a point on l_1 $(1, -1)$ and plug it into l_1 .

$$\begin{aligned}y &= x + b_1 \\-1 &= 1 + b_1 \\-2 &= b_1\end{aligned}$$

Now we have the y-intercept and the equation of the line l_1 is

$$\begin{aligned}l_1 : y &= m_1x + b_1 \\y &= x - 2\end{aligned}$$

Let's find the complete equation for the line l_2 . We already have that the slope is $m_2 = -2$. One point on line l_2 is $(x_0, y_0) = (-1, 2)$. Let's use this point to find the y-intercept.

$$\begin{aligned}y &= -2x + b_2 \\2 &= -2(-1) + b_2 \\ \therefore 0 &= b_2\end{aligned}$$

Now we have the equation for line l_2 as,

$$l_2 : y = m_2x + b_2$$

$$y = -2x + 0$$

$$y = -2x$$

Exercises

1. What is the slope of each line?

a) $y = -2x$

b) $y = x + 6$

c) $y = -\frac{1}{2}x + 4$

d) $y = \frac{5}{3}x - 5$

e) $y = -7x + 10$

2. What is the y-intercept for each line above?

3. Write the equation of the line with the following slope m and y-intercept b .

a) $m = -2, b = 0$

b) $m = -\frac{1}{2}, b = 1$

c) $m = 3, b = -5$

d) $m = \frac{1}{4}, b = \frac{7}{2}$