Scalar Equation of a Line

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Scalar or Cartesian equation of a line in $\mathbb{R}^2$

If we are given a vector $\underline{n} = (A, B)$ perpendicular to the line l then the scalar or Cartesian equation of the line is given by
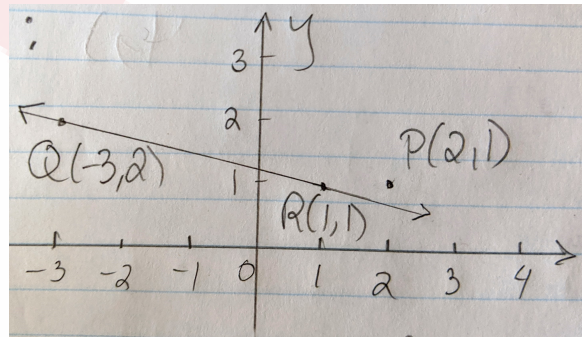
$$Ax + By + C = 0$$

We can take a point $P(x, y)$ on the line and plug it into the line to find the value for C.

Examples

Find the scalar equation of the line through P and in the direction of QR where P=(2, 1), Q = (-3, 2), R=(1, 1).

Solution:

Let's find the slope of the line.

$$m = \frac{rise}{run} = \frac{2 - 1}{-3 - 1} = \frac{1}{-4} \tag{1}$$

So far the equation of our line is,

$$y = mx + b = -\frac{1}{4}x + b$$

We need to find the y-intercept b. We know the point P=(2,1) lies on the on the line. So we can plug the point P into the equation (1) and then solve for b.

$$
\begin{aligned}
1 &= -\frac{1}{4}(2) + b \\
1 + \frac{1}{2} &= b \\
\frac{3}{2} &= b \\
\therefore y &= -\frac{1}{4}x + \frac{3}{2}.
\end{aligned}
$$

The scalar equation of the line is,

$$
\begin{aligned}
y &= -\frac{1}{4}x + \frac{3}{2} \\
4y &= 4\left(-\frac{1}{4}\right)x + 4\left(\frac{3}{2}\right) \\
4y &= -x + 6 \\
\therefore \quad & x + 4y - 6 = 0
\end{aligned}
$$

is the scalar equation of the line.

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Exercises

Given the points P, Q and R below, find the scalar equation of the line through the point P and perpendicular to the direction QR.

a) $P(3, 2), Q(0, 2), R(-3, 1)$

i) $P(-3, -4), Q(2, 0), R(5, 4)$

b) $P(4, 1), Q(-1, 0), R(0, 4)$

j) $P(0, -4), Q(3, 0), R(-2, 3)$

e) $P(-5, 3), Q(0, 1), R(5, 4)$

k) $P(-2, 5), Q(2, 2), R(1, 3)$

f) $P(-2, 0), Q(1, 3), R(4, -1)$