Vector and Parametric Equations of a Plane

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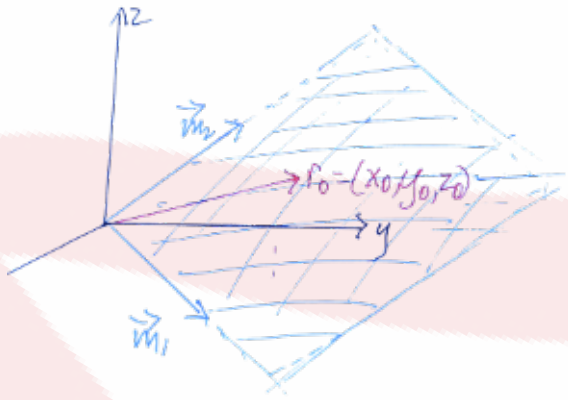
Vector and Parametric Equations of a Plane

Recall, for a line we needed one direction vector to determine what direction the line was point in. Since a plane is a 2-dimensional object, it requires two direction vectors to determine where it is located in $\mathbb{R}^3$. Let's call these two direction vectors for the plane $\overrightarrow{m}$ and $\overrightarrow{n}$.



Recalll for aline we also requied one point the line passes through. We ned that one point that lies on teh plane as well, $r_0 = (x_0, y_0, z_0)$. Now, the *vector equation of a plane* in $\mathbb{R}^3$ with direction vectors $\overrightarrow{m} = (m_1, m_2, m_3)$ and $\overrightarrow{n} = (n_1, n_2, n_3)$ and the point lying on the plane by $r_0 = (x_0, y_0, z_0)$ is given by,

$$\begin{aligned} \overrightarrow{r} &= \overrightarrow{r_0} + t\overrightarrow{n} + s\overrightarrow{m},\ s, t, \in \mathbb{R} \\ (x, y, z) &= (x_0, y_0, z_0) + t(n_1, n_2, n_3) + s(m_1, m_2, m_3). \end{aligned}$$

The *parametric equation of a plane* is $\mathbb{R}^3$ is,

$$\begin{aligned} x &= x_0 + tn_1 + sm_1 \\ y &= y_0 + tn_2 + sm_2 \\ z &= z_0 + tn_3 + sm_3,\ t, s \in \mathbb{R} \end{aligned}$$

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Exercises

1. State whether the following equations define a line or a plane. Justify your answer.
   (a) $\overrightarrow{r} = (1, 2, 3) + s(1, 1, 0) + t(3, 4, -6),\ s, t, \in \mathbb{R}$
   (b) $\overrightarrow{r} = (-2, 3, 0) + m(3, 4, 7),\ m \in \mathbb{R}.$
   (c) $x = -3 - t, y = 5, z = 4 + t,\ t \in \mathbb{R}$
   (d) $\overrightarrow{R} = m(4, -1, 2) + t(4, -1, 5),\ m, t, \in \mathbb{R}$

2. A plane passes through the points P(-2, 3, 1), Q(-2, 3, 2), and R(1, 0, 1). Using $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ as direction vectors, wrtie a vector equation for this plane.

3. Explain why the equation $\overrightarrow{r} = (-1, 0, -1)+s(2, 3, -4)+t(4, 6, -8)$ where $s, t \in \mathbb{R}$, does not represent the equation of a plane. What does this equation represent?

4. Determine vector equations and the corresponding parametric equations of the plane with direction vectors $\overrightarrow{a} = (4, 1, 0)$ and $\overrightarrow{b} = (3, 4, -1)$ and passing through the point A(-1, 2, 7).

5. A plane has $\overrightarrow{r} = (-3, 5, 6)+s(-1, 1, 2)+v(2, 1, -3)$ where $s, v \in \mathbb{R}$, as its equation.
   (a) Give the equations of two intersecting lines that lie on this plane.
   (b) What point do these two lines have in common?

6. Determine the equation of the plane that contains the point P(-1, 2, 1) and the line $\overrightarrow{r} = (2, 1, 3) + s(4, 1, 5),\ s \in \mathbb{R}.$

7. (a) Determine two pairs of direction vector that can be used to represent the xy-plane in $\mathbb{R}^3$.

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(b) Write a vector and parametric equations for the xy-plane in $\mathbb{R}^3$.

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