Cartesian Equation of a Plane in $\mathbb{R}^3$

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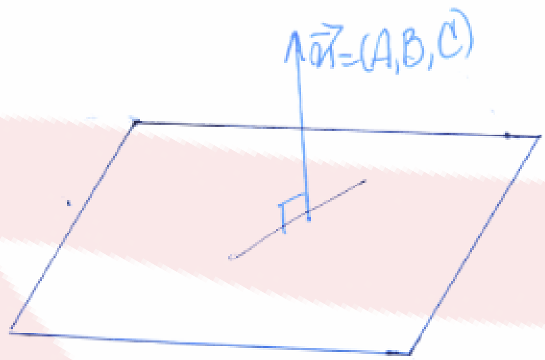
The *Cartesian equation of a plane* in $\mathbb{R}63$ is given by,

$$Ax + By + Cz + D = 0$$

where $\overrightarrow{n} = (A, B, C)$ is a vector perpendicular to the plane.



Example

Given points A(-1, 3, 8), B(-1, 1, 0) and C(4, 1, 1), find a poane through A, B and C and write the,

(a) parametric equation

(b) vector equation and

(c) Cartesian equation of the plane.

Solution: In order to determine the parametric and vector equation, we need to two direction vectors. To find the Cartesian equation, we need a normal to the plane. let's start by finding the two direction vectors $\overrightarrow{n}$ and $\overrightarrow{m}$.

$$\begin{aligned} \overrightarrow{n} &= A - B \\ &= (-1,3,8) - (-1,1,0) \\ &= (-1+1, 3-1, 8-0) \\ &= (0,2,8) \end{aligned}$$

and,

$$\begin{aligned} \overrightarrow{m} &= A - C \\ &= (-1,3,8) - (4,1,1) \\ &= (-1-4, 3-1, 8-1) \\ &= (-5,-2,7) \end{aligned}$$

To find the normal to the plane we can find the cross product of $\overrightarrow{n}$ and $\overrightarrow{m}$.

$$\begin{aligned} \overrightarrow{n} \times \overrightarrow{m} &= \begin{vmatrix} hati & \hat{j} & \hat{k} \\ 0 & 2 & 8 \\ -5 & -2 & 7 \end{vmatrix} \\ &= \hat{i}(14+16) - \hat{j}(0+40) + \hat{k}(0+10) \\ &= 30\hat{i} - 40\hat{j} + 10\hat{k} \\ &= (30,-40,10) \\ &= 10(3,-4,1) \end{aligned}$$

Therefore, we can use either $(30,-40,10)$ or $(3,-4,1)$ as the normal.

(a) The parametric eqation of the plane is given by,

$$\begin{aligned} x &= -1 - 5s \\ y &= 3 + 2t - 2s \\ z &= 8 + 8t + 7s \end{aligned}$$

(b) The vector equation of the plane is given by,

$$\overrightarrow{r} = (-1, 3, 8) + t(0, 2, 8) + s(-5, -2, 7)$$

(c) To find the Cartesian equation of the plane we need to plug a point say A(-1, 3, 8) into $30x - 40y + 10z + C = 0$. This gives,

$$\begin{aligned} 30x - 40y + 10z + C &= 0 \\ 30(-1) - 40(3) + 10(8) + C &= 0 \\ -30 - 120 + 80 + C &= 0 \\ -70 + C &= 0 \\ \therefore C &= 70 \end{aligned}$$

Therefore, $30x = 40y + 10z + 70 = 0$ or $3x - 4y + z + 7 = 0$ is the Cartesian equation of the plane.

Exercises

1. A plane is defined by the equation $x - 7y - 18z = 0$.
   (a) What is a normal vector to this plane?
   (b) Explain how you know that this plane passes through the origin.
   (c) Write the coordinates of three points on this plane.

2. A plane is defined by the equation $x = 0$.
   (a) What is a normal vector to this plane?
   (b) Explain how you know that this plane passes through the origin.
   (c) Write the coordinates of three points on this plane.

3. (a) A plane is determine by a normal $\overrightarrow{n} = (15, 75, -105)$ and passes through the origin. Write the Cartesian equation of this plane where the normal is in reduced form.
   (b) A plane has a normal of $\overrightarrow{n} = \left(-\frac{1}{2}, \frac{3}{4}, \frac{7}{16}\right)$ and passes through the origin. Determine the Cartesian equation of this plane.

4. Determine the Cartesian equation of the plane that contains the points A(-2, 3, 1), B(3, 4, 5) and C(1, 1,0).

5. The line with vector equation $\overrightarrow{r} = (2, 0, 1) + s(-4, 5, 5)$ where $s \in \mathbb{R}$, lies on the plane π, as does the point P(1, 3, 0). Determine the Cartesian equation of π.

6. (a) How would you find the angle formed between two intersecting planes.
   (b) Determine the angle between the planes $x - z + 7 = 0$ and $2x + y - z + 8 = 0$.

7. (a) Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$.

   (b) Determine the Cartesian equation of the plane that passes through the point P(1, 2, 1) and is perpendicular to the line,

$$\frac{x-3}{-2} = \frac{y+1}{3} = \frac{z+4}{1}$$

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8. (a) What is the value of k that makes the planes $4x+ky-2z+1 = 0$ and $2x + 4y - z + 4 = 0$ parallel?

   (b) What is the value of k that makes these two planes perpendicular?

   (c) Can these two planes ever be coincident? Explain.