Vector, Parametric, Symmetric Equations of a Line in $\mathbb{R}^3$

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2021

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For a point $r_o = (x_0, y_0, z_0) \in \mathbb{R}^3$ and a direction vector $\overrightarrow{m} = (a, b, c) \in \mathbb{R}^3$ the *vector equation* of a line through r_0 in the direction of $\overrightarrow{m}$ is,

$$\overrightarrow{r} = \overrightarrow{r_0} + t\overrightarrow{m},\ t \in \mathbb{R}$$

and the *parametric equation* of the lines is,

$$\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \\ z &= z_0 + tc \end{aligned}$$

where $\overrightarrow{r} = (x, y, z)$ is a point on the line and $t \in \mathbb{R}$.

The *symmetric equation* of the line is given by,

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

where $a, b, c \neq 0$.

Example

Given the points A(-1, 5, 7) and B(3, -4, 8) find,

(a) the vector equation of the line

(b) the parametric equation of the line

(c) the symmetric equation of the line

Solution: To find any of these equations we need a direction vector for the line. The direction vector $\overrightarrow{m}$ is,

$$\begin{aligned} \overrightarrow{m} &= \overrightarrow{OA} - \overrightarrow{OB} = (-1, 5, 7) - (3, -4, 8) \\ &= (-1 - 3, 5 + 4, 7 - 8) \\ &= (-4, 9, -1) \end{aligned}$$

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(a) $\overrightarrow{r} = (-1, 5, 7) + t(-4, 9, -1)$

(b)

$$\begin{aligned} x &= -1 - 4t \\ y &= 5 + 9t \\ z &= 7 - t \end{aligned}$$

where $t \in \mathbb{R}$.

(c)

$$\frac{x+1}{-4} = \frac{y-5}{9} = \frac{z-7}{-1}$$

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Exercises

1. State the coordinate of a point on each of the given lines.
   (a) $\overrightarrow{r} = (-3, 1, 8) + s(-1, 1, 9),\ s \in \mathbb{R}$
   (b) $x = -2 + 3t, y = 1 + (-4t), z = 3 - t,\ t \in \mathbb{R}$
   (c) $\frac{x+2}{-1} = \frac{z-1}{2}, y = -3$
2. State the direction vector for each line below,
   (a) $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-1}$
   (b) $x = 3, y = -2, z = -1 + 2k,\ k \in \mathbb{R}$
   (c) $\frac{x=1/3}{1/2} = \frac{y+3/4}{-1/4} = \frac{z-2/5}{1/2}$
3. A line passes through the points A(-1, 2, 4) and B(3, -3, 5).
   (a) Write two vector equation for this line,
   (b) Write the two sets of parametric equations associated with the vector equations in part (a).
4. A line passes through the points A(-1, 5, -4) and B(2, 5, -4).
   (a) Write a vector equation for the line containing these points.
   (b) Write parametric equations corresponding to the vector equation you wrote in part (a).
   (c) Explain why there ar eno symmetric equations for this line.
5. (a) Determine parametric equations for each of the following lines,

$$\frac{x+6}{1} = \frac{y-10}{-1} = \frac{z-7}{1}$$

   and

$$\frac{x+7}{1} = \frac{y-11}{-1}, z = 5$$

   (b) Determine the angle between the two lines.

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6. Determine the value of k for which the direction vectors of the lines,

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z+1}{k-1} \text{ and } \frac{x+3}{-2} = \frac{z}{1}, y = -1$$

are perpendicular.

7. (a) Write $(x, y, z) = (4, -2, 5) + t(-4, -6, 9)$ in parametric form.
   (b) Write $x = -4 + 5x, y = 2 - s, z = 9 - 6s$ in symmetric form.
   (c) Write $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z}{4}$ in vector form.
   (d) Write $x = -4, \frac{y-2}{3} = \frac{z-3}{5}$ in parametric form.

8. Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines,

$$\frac{x}{-4} = \frac{y+10}{-7} = \frac{z+2}{3}$$

and

$$\frac{x-5}{3} = \frac{y-5}{2} = \frac{z+5}{4}$$

and passes through the point (2, -5, 0).

9. Determine the angle formed by the intersection of the lines defined by,

$$\frac{x-1}{2} = \frac{y+3}{1}, z = -3$$

and

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{z}{1}$$

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