Vector and Parametric Equations of a Line in $\mathbb{R}^2$

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Given a point r_0 and a vecctor $\overrightarrow{m}$ in $\mathbb{R}^2$ the *vector equation* passing through the point in the directon $\overrightarrow{m}$ is given by,

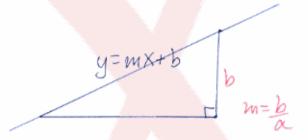$$\overrightarrow{r} = \overrightarrow{r}_0 + t\overrightarrow{m}$$

where $t \in \mathbb{R}$ and $\overrightarrow{r_0}$ is the position vector through the point r_0.

Letting $\overrightarrow{r} = (x,y)$ and $\overrightarrow{m} = (m_1, m_2)$ and $\overrightarrow{r_0} = (x_0, y_0)$ the *parametric equation of the line* passing through r_0 is the direction $\overrightarrow{m}$ is given by,

$$\begin{aligned} x &= x_0 + tm_1 \\ y &= y_0 + tm_2, \quad \text{where } t \in \mathbb{R}. \end{aligned}$$

The value t is called a *parameter* and can take on any real number in value.

The form of a line $y = mx + b$ is familiar to use. m is the slope and b the y-intercept of the line. How is the slope related to the direction vector, if at all, of a line in vector form? Good question!

Our slope is defined as $\frac{rise}{run}$ or from the diagram $\frac{b}{a}$. This means, we can use the vector $\overrightarrow{m} = (a, b)$ to describe the direction of any line parallel to our given line. This line passes through the point $(0, b)$ since b is the y-intercept. So now, if we let, $\overrightarrow{r_0} = (0, b)$ and $\overrightarrow{m} = (a, b)$ then we have the vector form of the line given by,

$$\overrightarrow{r} = (0, b) + t(a, b), \quad \text{where } t \in \mathbb{R}$$

and parametric form given by,

$$\begin{aligned} x &= ta \\ y &= b + tb, \quad \text{where } t \in \mathbb{R} \end{aligned}$$

Exercises

1. A line passes through the points A(2,1) and B(-3,5). Write two different vector equations for this line.

2. A line is defined by the parametric equations $x = -2 - t$ and $y = 4 + 2t$, $t \in \mathbb{R}$.

   (a) Does R(-9,18) lie on this line? Explain.

   (b) Write a vector equation for this line using the given parametric equations.

3. (a) If the equation of a line if $\overrightarrow{r} = s(3,4)$, $s \in \mathbb{R}$, name the coordinates of three points on this line.

   (b) Write a vector equation, different from the one given, in part (a) that also passes through the origin.

   (c) Describe how the line with equation $\overrightarrow{r} = (9,12) + t(3,4)$, $t \in \mathbb{R}$ related to the line given in part (a).

4. A line passes through the points Q(0,7) and R(0,9).

   (a) Sketch this line.

   (b) Determine vector and parametric equations for this line.

5. For the line L: $\overrightarrow{r} = (1,-5) + s(3,5)$, $s \in \mathbb{R}$, determine the following,

   (a) an equation for the line perpendicular to L passing through P(2,0).

   (b) the point at which the line in part (1) intersects the y-axis.

6. Are the lines $2x - 3y + 15 = 0$ and $(x, y) = (1,6) + t(6,4)$ parallel? Explain.

7. What is the direction vector and a point on each of the following lines,

(a) $\vec{r} = (3, 4) + t(2, 1), \ t \in \mathbb{R}$

(b) $x = 1 + 2t, y = 3 - 7t, \ t \in \mathbb{R}$

(c) $\vec{r} = (4, 1 + 2t), \ t \in \mathbb{R}$

(d) $x = -5t, y = 6, \ t \in \mathbb{R}$