Properties of Vectors

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For non-zero vectors $\overrightarrow{w}$, $\overrightarrow{v}$ and $\overrightarrow{u}$ and scalrs $k, m, n \in \mathbb{R}$ we have the following properties:

1. $\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}$ (Commutative Property)
2. $(\overrightarrow{u} + \overrightarrow{v}) + \overrightarrow{w} = \overrightarrow{u} + (\overrightarrow{v} + \overrightarrow{w})$ (Associative Property)
3. $k(\overrightarrow{u} + \overrightarrow{v}) = k\overrightarrow{u} + k\overrightarrow{v}$ (Distributive Property)
4. $\overrightarrow{0} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{0} = \overrightarrow{v}$
5. $m(n\overrightarrow{v}) = (mn)\overrightarrow{v} = mn\overrightarrow{v}$
6. $(m+n)\overrightarrow{v} = m\overrightarrow{v} + n\overrightarrow{v}$

Exercises

1. Write the following vector in simplified form: $3(\overrightarrow{a} - 2\overrightarrow{b} - 5\overrightarrow{c}) - 3(2\overrightarrow{a} - 4\overrightarrow{b} + 2\overrightarrow{c}) - (\overrightarrow{a} - 3\overrightarrow{b} + 3\overrightarrow{c})$
2. If $\overrightarrow{a} = 3\overrightarrow{i} - 4\overrightarrow{j} + \overrightarrow{k}$ and $\overrightarrow{b} = -2\overrightarrow{i} + 3\overrightarrow{j} - \overrightarrow{k}$, express each of the following in terms of $\overrightarrow{i}$, $\overrightarrow{j}$ and $\overrightarrow{k}$.
3. If $2\overrightarrow{x} + 3\overrightarrow{y} = \overrightarrow{a}$ and $-\overrightarrow{x} + 5\overrightarrow{y} = 6\overrightarrow{b}$, express $\overrightarrow{x}$ and $\overrightarrow{y}$ in terms of $\overrightarrow{a}$ and $\overrightarrow{b}$.
4. If $\overrightarrow{x} = \frac{2}{3}\overrightarrow{y} + \frac{1}{3}\overrightarrow{z}$, $\overrightarrow{x} - \overrightarrow{y} = \overrightarrow{a}$ and $\overrightarrow{y} - \overrightarrow{z} = \overrightarrow{b}$, show that $\overrightarrow{a} = -\frac{1}{3}\overrightarrow{b}$.
5. Two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ have a common starting point with an angle of 12-° between them. The vectors are such that $|\overrightarrow{a}| = 3$ and $|\overrightarrow{b}| = 4$.
   (a) Calculate $|\overrightarrow{a} + \overrightarrow{b}|$.
   (b) Calculate the angle between $\overrightarrow{a}$ and $\overrightarrow{a} + \overrightarrow{b}$.
6. Determine all possible values for t if the length of the vector $\overrightarrow{x} = t\overrightarrow{y}$ is $4|\overrightarrow{y}|$.
7. PQRS is a quadrilateral with $\overrightarrow{PQ} = 2\overrightarrow{a}$, $\overrightarrow{QR} = 3\overrightarrow{b}$ and $\overrightarrow{QS} = 3\overrightarrow{b} - 3\overrightarrow{a}$. Express $\overrightarrow{PS}$ and $\overrightarrow{RS}$ in terms of $\overrightarrow{a}$ and $\overrightarrow{b}$.
8. An airplane is heading due south at a speed of 500km/h when it encounters a head wind fro the south at 40km/h. What is the resultant ground veloity of the airplane?
9. If A B and C are three collinear points with B at the midpoint of AC and O is any point not on the line AC, prove that $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$. (Hint: $\overrightarrow{AB} = \overrightarrow{BC}$)

10. ABCD is aparallelogram. If $\overrightarrow{AB} = \overrightarrow{x}$ and $\overrightarrow{DA} = \overrightarrow{y}$, express $\overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{BD}$ and $\overrightarrow{AC}$ in terms of $\overrightarrow{x}$ and $\overrightarrow{y}$.

11. The vector $\overrightarrow{m}$ is collinear (parallel) to $\overrightarrow{b}$ but in the opposite direction. Express the magnitude of $\overrightarrow{m} + \overrightarrow{n}$ in terms of the magnitudes of $\overrightarrow{m}$ and $\overrightarrow{n}$.

12. The vectors $\overrightarrow{p}$ and $\overrightarrow{q}$ are distinct unit vectors that are placed in a tail-to-tail position. If these two vectors have an angle of $60°$ between them, determine $|2\overrightarrow{p} - \overrightarrow{q}|$.

13. Given that $|\overrightarrow{u}| = 8$ and $|\overrightarrow{v}| = 10$ and the angle between vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ is $60°$ determine:
    (a) $|\overrightarrow{u} - \overrightarrow{v}|$
    (b) the direction of $\overrightarrow{u} - \overrightarrow{v}$ relative to $\overrightarrow{u}$
    (c) the unit vector in the direction of $\overrightarrow{u} + \overrightarrow{v}$
    (d) $|5\overrightarrow{u} + 2\overrightarrow{v}|$

