Vector Addition

RaiseMyMarks.com

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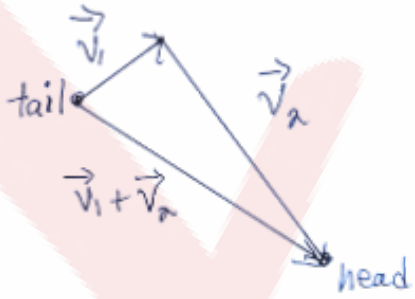
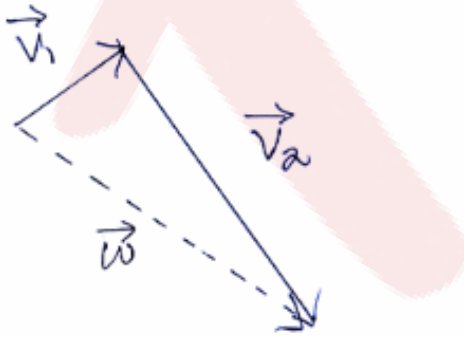
Vector Addition

Vector addtion is easier to do and explain visually. Say we have two vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ given below,

vectors is performed by lining up the vectors head to tail and drawing in the *resultant* vector from the orginal tail to the final head. Let's go through this process with vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$.

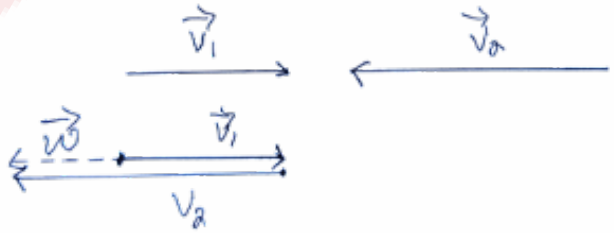


resultant $= \overrightarrow{v_1} + \overrightarrow{v_2}$



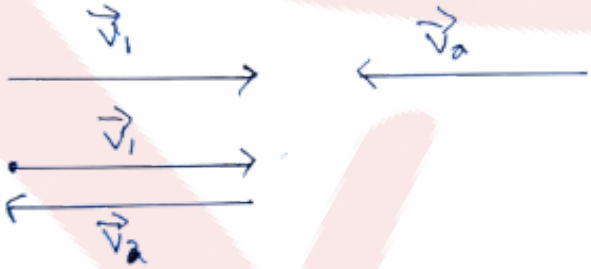
$\overrightarrow{w} = \overrightarrow{v_1} + \overrightarrow{v_2}$ = resultant vector

Let's consider a few more examples.



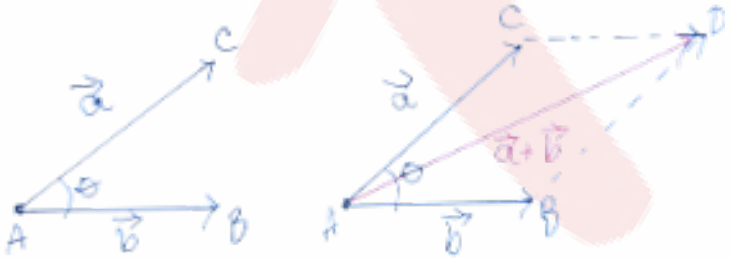
$\overrightarrow{w} = \overrightarrow{v_1} + \overrightarrow{v_2}$ =resultant vector

Opposite vectors



$\overrightarrow{w} = \overrightarrow{v_1} + \overrightarrow{v_2} = \overrightarrow{0}$ = zero vector

The paralellogram law for adding two vectors



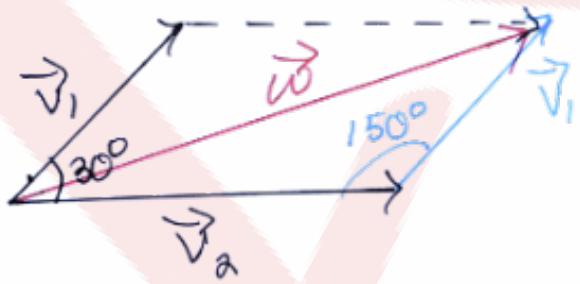
To determine the sum of two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ complete the parallelograpm formed by $\overrightarrow{a}$ and $\overrightarrow{b}$. The sum is the diagonal of the parallelogram, $\overrightarrow{AD}$. Thererfore,

$$\overrightarrow{a} + \overrightarrow{b} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$$

Example

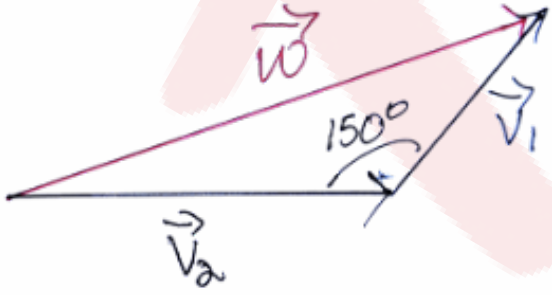
Given two vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ with an angle of 30° between them, $|\overrightarrow{v_1}| = 3$ and $|\overrightarrow{v_2}| = 4$, determine $|\overrightarrow{v_1} + \overrightarrow{v_2}|$.

Solution We start by using the parallelogram law to find the sum of $\overrightarrow{v_1} + \overrightarrow{v_2}$.



$\overrightarrow{w} = \overrightarrow{v_1} + \overrightarrow{v_2}$.

Now we can use the triangle,



formed by the parallelogram law to find the magnitude of $|\overrightarrow{w}|$. We know $|\overrightarrow{v_1}| = 3$ and $|\overrightarrow{v_2}| = 4$ and the angle 150°, we can use the cosine law to find $|\overrightarrow{w}|$.

$$\begin{aligned} |\overrightarrow{w}| &= |\overrightarrow{v_1}|^2 + |\overrightarrow{v_2}|^2 - 2|\overrightarrow{v_1}||\overrightarrow{v_2}| \cos 150^\circ \\ &= 3^2 + 4^2 - 2(3)(4) \cos 150^\circ \\ &= 9 + 16 - 24\left(-\frac{\sqrt{3}}{2}\right) \\ &= 25 + 12\sqrt{3} \\ &\approx 45.78 \end{aligned}$$

Scalar Multiplication of a Vector

For a positive scalar, a number, $a > 0$, and vector $\overrightarrow{v}$, $a\overrightarrow{v}$ is a vector in the direction of $\overrightarrow{v}$ with magnitude $|a||\overrightarrow{v}|$. When $a < 0$, a negative scalar, $a\overrightarrow{v}$ is a vector in the opposite direction of $\overrightarrow{v}$ with magnitude $|a||\overrightarrow{v}|$.

Example

Given the vector $\overrightarrow{v}$ what is the reulstant vector for the following,

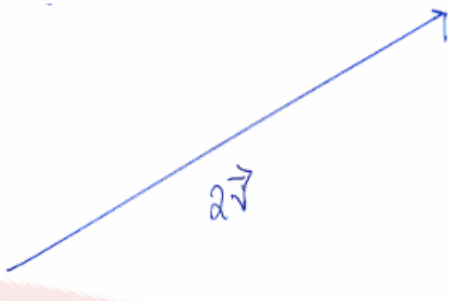
(a) $-\overrightarrow{v}$

(b) $2\overrightarrow{v}$

Solution

(a)

(b)



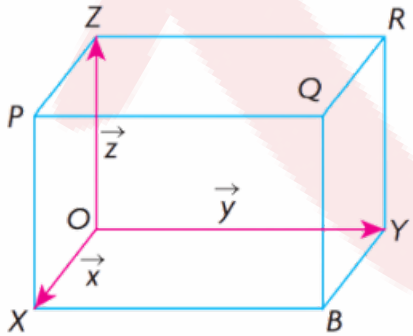
Exercises

1. The vectors $\overrightarrow{x}$ and $\overrightarrow{y}$ are drawn as shown below. Draw a vector equivalent to each of the following.

   (a) $\overrightarrow{x} + \overrightarrow{y}$
   (b) $\overrightarrow{x} - \overrightarrow{y}$
   (c) $\overrightarrow{y} - \overrightarrow{x}$
   (d) $-\overrightarrow{y} + (-\overrightarrow{x})$

2. Each of the following vector expressions can be siplified and written as a single vector. Wrinte the single vector vector correspondingn to each expression and illustrate your answer with a sketch.
   (a) $\overrightarrow{PQ} - \overrightarrow{RQ} + \overrightarrow{RS}$
   (b) $\overrightarrow{PS} - \overrightarrow{RS} - \overrightarrow{PQ}$

3. The rectangular box shown below is labelled with $\overrightarrow{OX} = \overrightarrow{x}$, $\overrightarrow{OY} = \overrightarrow{y}$ and $\overrightarrow{OZ} = \overrightarrow{z}$.



Express each of the following vectors in terms of $\overrightarrow{x}$, $\overrightarrow{y}$ and $\overrightarrow{z}$.

(a) $\overrightarrow{BY}$

(b) $\overrightarrow{XB}$

(c) $\overrightarrow{OB}$

(d) $\overrightarrow{XY}$

(e) $\overrightarrow{OQ}$

(f) $\overrightarrow{QZ}$

(g) $\overrightarrow{XR}$

(h) $\overrightarrow{PO}$

4. In still water Maria can paddle at the rate of 7km/h. The current in which she paddles has a speed of 4 km/h.

   (a) At what velocity does she travel downstream?

   (b) Using vectors, draw a diagram that illustrates her velocity going downstream.

   (c) If Maria changes her direction and heads upstream instead, what is her speed? Using vectors, draw a diagram that illustrates her velocity going upstream.

5. A small airplane is flying due north at 150km/h when it encounters a wind of 80km/h from the east. What is the resultant ground velocity of the plane?

6.

$|\overrightarrow{x} = 7$ and $|\overrightarrow{y} = 24$. If the angle between these vectors is $90°$, determine $|\overrightarrow{x} + \overrightarrow{y}|$ and calculate the angle between $\overrightarrow{x}$ and $\overrightarrow{x} + \overrightarrow{y}$.

7. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two unit vectors (vectors with magnitude 1 with an angle of $150°$ between them. Calculate $|\overrightarrow{AB} + \overrightarrow{AC}|$.