Velocity

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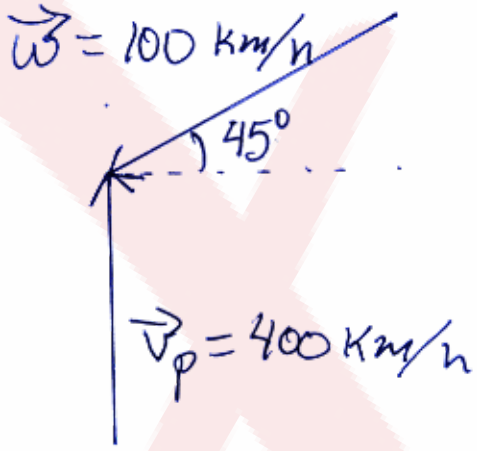
Velocity

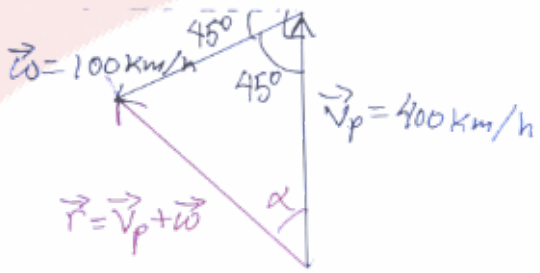
Velocity is another quantity that has a magnitude and direction. For example, a car travelling 60 km/h east has a velocity $\overrightarrow{v_c}$ of 60 km/h east and a speed of 60 km/h, $|\overrightarrow{v_c}|$. A bus travelling 60 km/h west has a velocity of $\overrightarrow{v_b}$ =60 km/h west and a speed of $|\overrightarrow{v_b}$ =60 km/h. Notice that, $\overrightarrow{v_b} = -\overrightarrow{v_c}$ and $|\overrightarrow{v_b}| = |\overrightarrow{v_c}|$.

Example

A plane is heading north with an air speed of 400 km/h. The plane is blown off course by a wind of 100 km/h *from* the north east. Determine the resultant ground velocity of the plane.

Solution:





We can use the cosine law to find the magnitude of the resultant vector.

$$\begin{aligned}
|\overrightarrow{r}|^2 &= |\overrightarrow{w}|^2 + |\overrightarrow{v}|^2 - 2|\overrightarrow{w}||\overrightarrow{v}|\cos(45^\circ) \\
&= 100^2 + 400^2 - 2(100)(400)\left(\frac{1}{\sqrt{2}}\right) \\
&= 10000 + 160000 - \frac{80000}{\sqrt{2}} \\
&= 170000 - \frac{80000}{\sqrt{2}} \\
&= 113431.4574 \\
\therefore |\overrightarrow{r}| &\approx 336.80
\end{aligned}$$

To find the direction we can use the sine law.

$$\begin{aligned}
\frac{\sin\alpha}{100} &= \frac{\sin 45^\circ}{336.80} \\
\sin\alpha &= \frac{100(1/\sqrt{2})}{336.80} \approx 0.2099 \\
\therefore \alpha &= 12.1^\circ
\end{aligned}$$

Therefore, the resulting velocity is 336.80 km/h and 12.1° north west.

Example

A river is 2 km wide and flows at 6 km/h. anna is driving a motorboat which has a speed of 20 km/h in still water and she heads out from

one bank in a direction perpendicular to the current. A marina lies directly across the river from the starting point on the opposite bank.

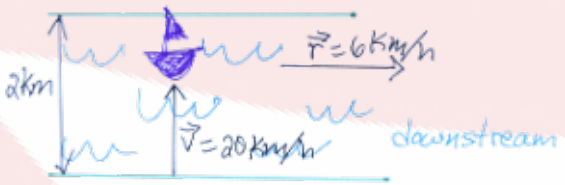
(a) How far downstream from the marina will the current push the boat?

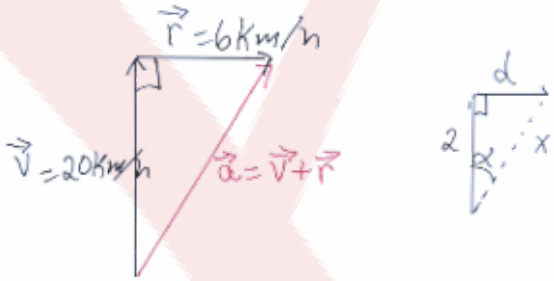
(b) How long will it take for the boat to cross the river?

(c) If Anna decides that she wants to end up directly across the river at the marina, in what direction should she head? What is the resultant velocity of the boat?

Solution:INCOMPLETE

(a)





$\overrightarrow{a} = \overrightarrow{v} + \overrightarrow{r}$.

$$\begin{aligned} \tan\theta &= \frac{6}{20} = \frac{3}{10} \text{ and} \\ |\overrightarrow{a}|^2 &= |\overrightarrow{v}^2 + |\overrightarrow{r}|^2 \\ &= 400 + 36 \\ &= 436 \\ \therefore |\overrightarrow{a}| &= \sqrt{436} \end{aligned}$$

Exercises

1. A woman walks at 4km/h down the corridor of a train that is travelling at 80km/h on a straight track.
   (a) What is her resultant velocity in relation to the group if she is walking in the same direction as the train?
   (b) If she walks in the opposite direction as the train, what is her resultant velocity?
2. An airplan heading north has an air speed of 600km/h.
   (a) If the airplan encounters a wind from the north at 100km/h, what is the resultant ground velocity of the plane?
   (b) If there is a wind from the south at 100km/h, what is the resultant ground velocity of the plane?
3. Adam can swim at the rate of 2km/h in still water. At what angle to the bank of the river must he head if he wants to swim directly across the river and the current in the river moves at the rate of 1 km/h?
4. A child, sittingn in the backseat of a car travelling at 20m/s, throws a ball at 2m/s to her brother who is sitting in the front seat.
   (a) What is the velocity of the ball relative to the children?
   (b) What is the velocity of the ball relative to the road?
5. An airplan is headed north with a constant velocity of 450km/h. The plane encounters a wind from the west at 100km/h.
   (a) In 3 hours how far will the plane travel?
   (b) In what direction will the plane travel?
6. Judy and her friend Helen live on opposite sides of a river that is 1km wide. Helen lives 2km downstream from Judy on the opposite

side of the river. Judy can swim at a rate of 3 km/h and the river's current has a speed of 4km/h. Judy swims from her cottage directly across the river.

(a) What is Judy's resultant velocity?

(b) How far away from Helen's cottage will Judy be when she reachees the other side?

(c) How long will it take Judy to reach the other side?

7. Barbara can swim at 4km/h in still water. She wishes to swim across a river to a point directly opposite from where she is standing. The river is moving at a rate of 5km/h. Explain using diagrams why this is not possible.

8. Dave wants to cross a 200m side river whose current flows at 5.5m/s. The marina he wants to visit is located at an angle of S45°W from his starting position. Dave can paddle his canoe at 4m/s in still water.

(a) In which direction should he head to get to the marina?

(b) How long will the trip take?

