Vectors as Forces

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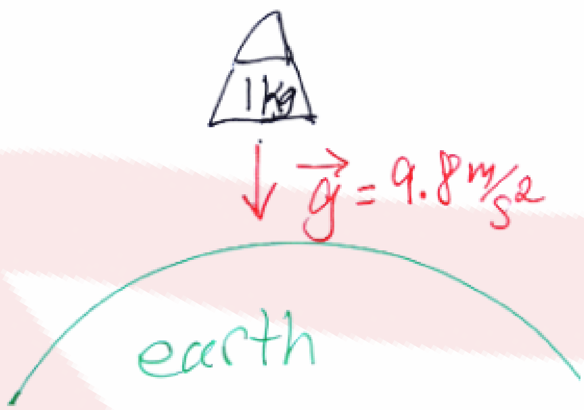
Vectors as Forces

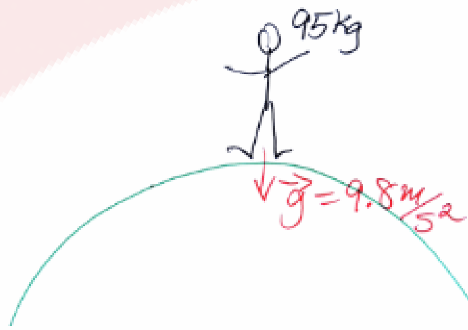
Force is something we are familar with and usually seen in a physics class. Force can be thought of as a push or pull in a given direction with a given magnitude and so, a force can be represented by a vector. What is the unit of measure for force? Let's consider an example.



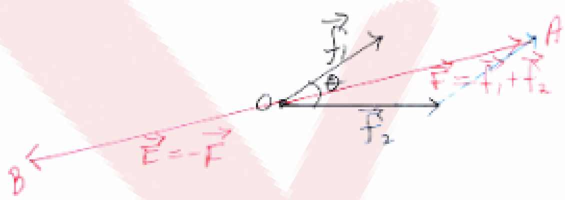
The acceleartion due to gravity is pulling a 1kg weight towards the centre of the earth with a force of,

$$\begin{aligned} & (1kg)(9.8m/s^2) \\ = \; & 9.8kgm/s^2 \text{ or } 9.8N \end{aligned}$$

where $N = kgm/s^2$ is a *Newton*. A *Newton* is the unit of measure of force. If we have a 95kg person standing on the surface of the earth, the force exerted on the person is $(95kg)(9.8m/s^2) = 931kgm/s^2 = 931N$ downwards.



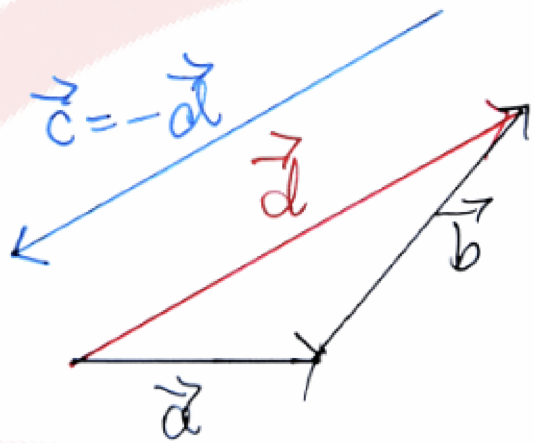
Usually an object has many forces acting on it in varying directions. The effect of all these forces or the composition of all these forces is a single foced called the *resultant* or *sum* of all these forces. The *equilibrant* of several forces is a single force that opposes the resultant of all the forces acting on an object. When the equilibrant is appliled to the object, this force maintains the object in a state of equilibrium.



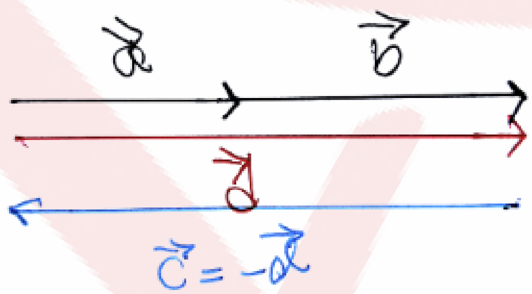
$\overrightarrow{OA} = \vec{F} = \vec{f_1} + \vec{f_2}$ is the resultant force.
$\overrightarrow{OB} = \vec{E} = -\vec{F}$ is the equilibrant force.

Forces in equilibrium

When 3 non-collinear forces, vectors, are in a state of equilibrium then these 3 vectors lie in the same plane.



$\vec{a}$, $\vec{b}$ are two non-collinear foces and $\vec{d}$ the resultant. $\vec{c} = -\vec{d}$ is the equilibrant force. $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar forces.

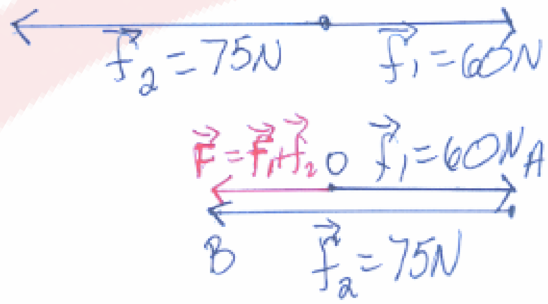


In this example above we have two collinear forces.

Example

Max the dog is pulling on a rope with a force of 60N east; the pup Angie is pulling on the rope in the opposite direction, west, with a force of 75N. What is the resultant and equilibrant of these two forces?

Solution:

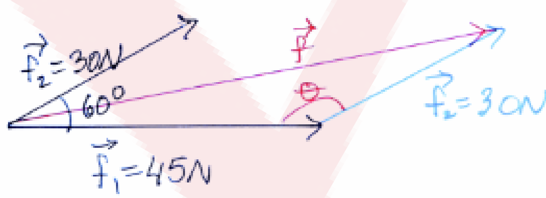


$\overrightarrow{F} = \overrightarrow{f_1} + \overrightarrow{f_2} = \overrightarrow{OB} = 15N$ west is the resultant force. The equilibrant force $\overrightarrow{E}$ is 15N east.

Example

Two forces of 30N and 45N act an angle of 60° to each other. What is the resultant of these two forces?

Solution:

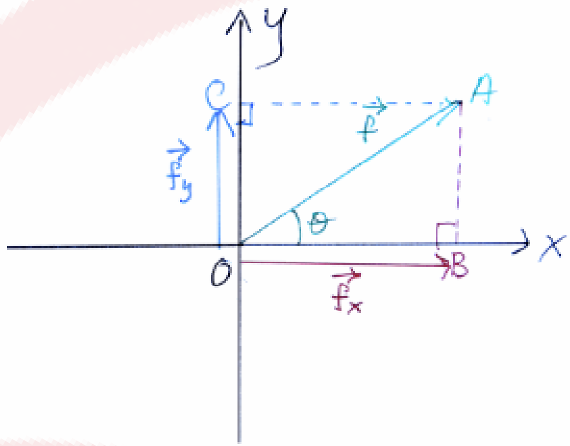


Above is a diagram describing the situation. We can use the cosine law to find our resulting for $f = |\overrightarrow{f}|$.

$$f^2 = f_1^2 + f_2^2 - 2f_1 f_2 \cos(120°)$$

Resolving a force into its components

A force can be resolved into its *components*. These components are the horizontal and vertical components in 2-dimensions. For example,



$\overrightarrow{f}$ is our given force and θ the angle it makes with the horizontal axis. From here we have,

$$f_x = |\overrightarrow{f}|\cos\theta$$
$$f_y = |\overrightarrow{f}|\sin\theta$$

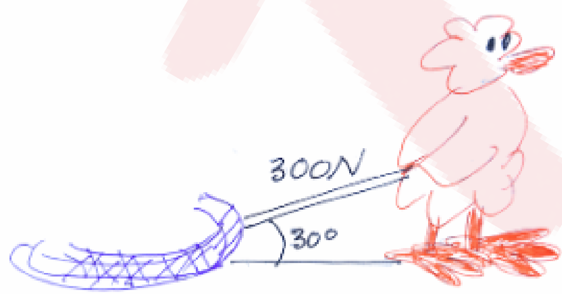
Example

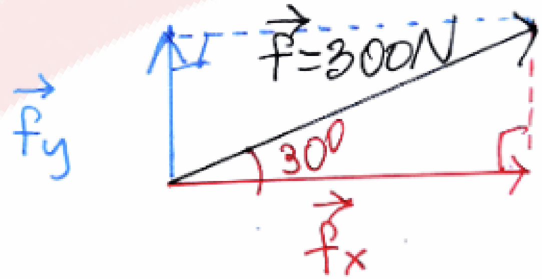
Big Bird pull son a roope attached to a sleigh with a force of 300N. If the rope makes an angle of 30° with the horizontal determine,

(a) the force that pulls the sleigh forward

(b) the force that pulls the sleigh vertically (lifts the sleigh upwards).

Solution:



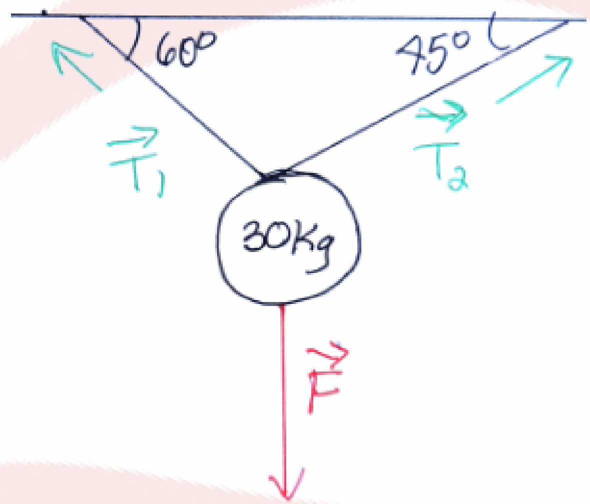


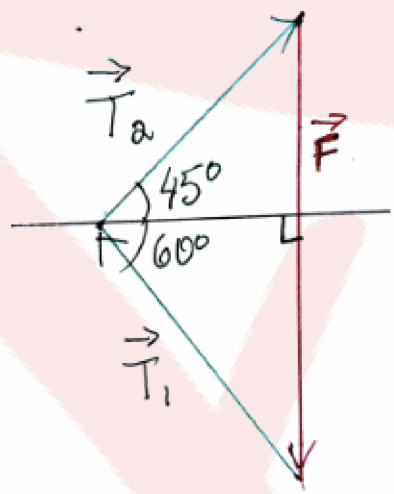
$$
\begin{aligned}
f_x &= |f|\cos\theta \\
&= 300\cos(30^\circ) \\
&= 300\frac{\sqrt{3}}{2} \\
&= 150\sqrt{3}N, \quad \text{and} \\
f_y &= |f|\sin\theta \\
&= 300\sin(30^\circ) \\
&= 300\left(\frac{1}{2}\right) \\
&= 150N
\end{aligned}
$$

Example

A mass of 30kg is hanging by two lengths of rope from the ceiling. The two ropes make angles of 60° and 45° with the ceiling. Determine the tension in each rope.

Solution: There are two different ways of of approaching this problem. Let's consider the first.



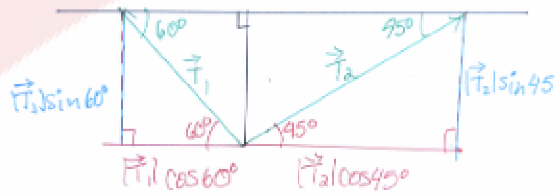


$$\begin{aligned} |f| &= \text{force due to gravity} \\ &= \text{mass } \times \text{ acceleration} \\ &= (30kg)(9.8m/s^2) \\ &= 294N \end{aligned}$$

We need $T_1 + T_2 = F$. We can use the sine law to find $|T_1|$ and $||T_2|$.

$$\frac{|T_2|}{\sin 30^\circ} = \frac{|T_1|}{\sin 45^\circ} = \frac{F}{\sin 105^\circ}$$

The second way of approaching this question,



For the system to be in equilibrium the horizontal and vertical components must balance each other out.

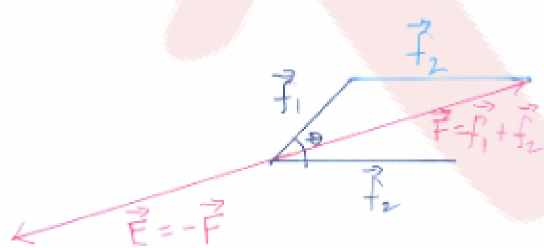
$$
\begin{aligned}
|T_1|\sin 60^\circ &= |T_2|\sin 45^\circ \\
|T_1|\cos 60^\circ &= |T_2|\cos 45^\circ
\end{aligned}
$$

Resultant and composition of froces

The resultant of several forces in the single force that can be used to represent the combined effect of all the forces. The individual forces that make up the resultant are referred to as the *components* of the *resultant*.

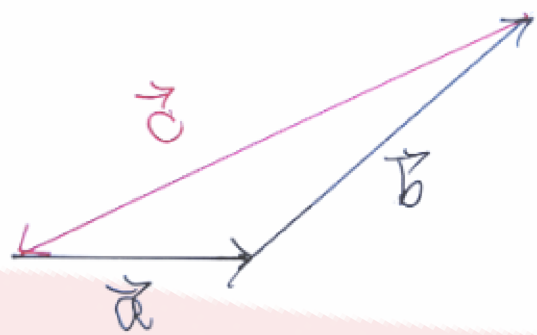
Equilibrant of several forces

The equilibrant of a number of forces is the single force that opposes the resultant of the forces acting on an object. When the equilibrant is applied to the object, this force maintains the object in a state of equilibrium.

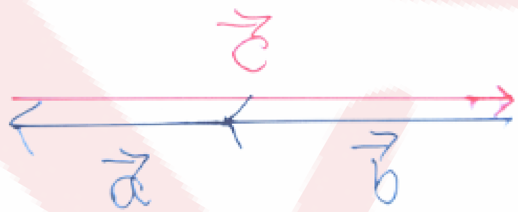


$E = -F$.

Forces in equilibrium



$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = \overrightarrow{0}$, three non-collinear forces.



$\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = \overrightarrow{0}$, three collinear forces.

Example

Is it possible for 3 fordes of 15N, 20N and 37N to keep a system in a state of equilibrium?

Solution: In order for three forces to be in equilibrium they need to form a triangle and satisfy the triangle inequality, $|a+b| \leq |a|+|b|$. But, we have $15+20 < 37$ which does not satisfy the triangular inequaility. so no, the three forces 15N, 20N and 37N cannot be in equilibrium.

Exercises

1. Three forces of 10N, 20N and 30N are in a state of equilibrium.
   (a) Draw a sketch of these three forces.
   (b) What is the angle between the equilibrant and each of the smaller forces?

2. Determine the resultant and equilibrant of each pair of forces acting on an object.
   (a) $\overrightarrow{f_1}$ has a magnitude of 5N acting due east and $\overrightarrow{f_2}$ has a magnitude of 12N acting due north.
   (b) $\overrightarrow{f_1}$ has a magnitude of 9N acting due west, and $\overrightarrow{f_2}$ has a magnitude of 12N acting due south.

3. A force $\overrightarrow{f_1}$ of magnitude 6N acts on a particle P. A second force $\overrightarrow{f_2}$ of magnitude 8N acts at 60° to $\overrightarrow{f_1}$. Determine the resultant and equilibrant of $\overrightarrow{f_1}$ and $\overrightarrow{f_2}$.

4. Resolve a force of 10N into two forces perpendicular to each other, such that one component force makes an angle of 15° with the 10N force.

5. A 10kg block lies on a smooth ramp that is inclined at 30°. What force parallel to the ramp would prevent the block from moving?

6. Four forces of magnitude 5N, 9N, 10N and 14N are a rranged as shown in the diagram at the left. Determine the resultant of these forces.

7. Three forces each having a magnitude of 1N are arranged to produce equilibrium.

(a) Draw a sketh showing an arrangement of these forces and show that the angle between the resultant and each of the other two forces is 60°.

(b) Explain how to determine the angle between the equilibrant and the tother two vectors.

8. A mass of 20kg is suspended from a ceiling by two lengths of rope that make angles of 30° and 45° with the ceiling. Determine the tension in each of the ropes.

9. Two tug boats are towing a ship. The smaller tug boat is 15° off the port bow and the larger tug boat is 20° off the starboard bow. The larger tug boat pulls twice as hard as the smaller tug boat. In what direction will the ship move?

10. Three forces of 5N, 8N and 10N act from the corner of the rectangular solid along its three edges.

    (a) Calculate the magnitude of the equilibrant of these three forces.

    (b) Determine the angle that the equilibrant makes with each of the three forces.