Applications of the dot and cross product

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Many applications of the dot and cross product can be found in physics and geometry. Below are some ways the dot or corss product can be applied to physical situations.
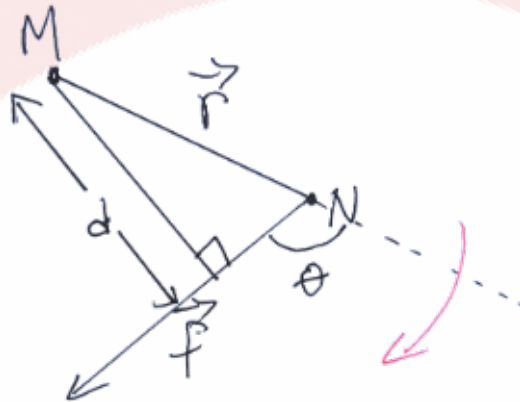
1. Given $\overrightarrow{F}$ is the force applied to an object in Newtons and $\overrightarrow{s}$ the displacement of the object in metres then the work in Jourles is give by $w = \overrightarrow{F} \cdot \overrightarrow{s}$.

2. For vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ the area of the parallelogram formed by $\overrightarrow{a}$ and $\overrightarrow{b}$ is given by,

$$\text{area} = |\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{a}||\overrightarrow{b}|\sin\theta$$
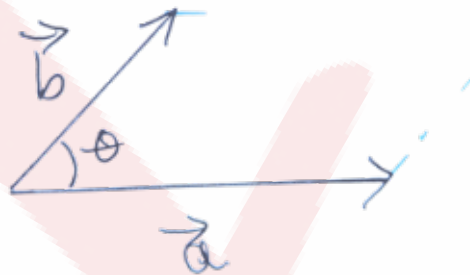
3. From 2. we have that the area of a triangle with sides $\overrightarrow{a}$ and $\overrightarrow{b}$ is given by,

$$\text{area of triangle} = \frac{1}{2}|\overrightarrow{a} \times |overrightarrowb|$$

4.

For a force $\overrightarrow{f}$ applied to a lever $\overrightarrow{r}$, rotating about a point M where $\overrightarrow{r} = \overrightarrow{MN}$, the *torque* or twisting effect of this force about m is given by,

$$|\tau| = |\overrightarrow{r} \times \overrightarrow{f}| = |\overrightarrow{r}||\overrightarrow{f}|\sin\theta.$$

Exercises

1. (a) Calculate $|\vec{a} \times \vec{b}|$ where $\vec{a} = (1, 2, 1)$ and $\vec{b} = (2, 4, 2)$.

   (b) If $\vec{a}$ and $\vec{b}$ represent the sides of a parallelograph, explain why the answer in part (a) makes sense in terms of the formula for the area of a parallelogram.

2. Calculate the amount work done when

   (a) a 40kg rock falls 40m down a slope at an angle of $50°$ to the vertical.

   (b) a lawnmower is pushed 500m by a force of 100N applied at an angle of $45°$ to the horizontal.

3. Calculuate the are of the parallelogram formed by the following pairs of vectors

$$\vec{a} = (1, -2, 3) \text{ and } \vec{b} = (1, 2, 4).$$

4. The area of the parallelogram formed by the vectors $\vec{p} = (a, 1, -1)$ and $\vec{q} = (1, 1, 2)$ is $\sqrt{35}$. Determine the value(s) of a for which tis is true.

5. in $\mathbb{R}^3$ points A(-2, 1, 3), B(1, 0, 1) and C(2, 3, 2)form the vertices of $\triangle ABC$.

   (a) By constructiong position vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, determine the area of the triangle.

   (b) By constructiong position vectors $\overrightarrow{BC}$ and $\overrightarrow{CA}$, determine the area of the triangle.

   (c) What conclusion can be drawn?

6. A 10N force is applied at the end of the wrench that is 14cm long. The force makes an angle of $45°$ with the wrench. Determine the

magnitude of the torque of this force about the other end of the wrench.