The Intersection of Planes

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The Intersection of Two Planes

Line of intersection

Plane of intersection

Intersection of Three Planes

Point of intersection

0.1   Line of Intersection

Plane of Intersection

Since the equations of lines and planes are all "linear", the above situations all result in a consistent linear system. What does the situation look like graphically when the linear system is inconsistent?

Inconsistent Systems

Two Planes

Two planes create an inconsistent system when the planes are parallel.

Three Planes

Three planes create an inconsistent system when at least two of the planes are parallel.

The other case when three planes create an inconsisten system is when the three planes create a *trianglar prism*.

Exercises

1. Find the solution to the given system of equations using elementary row operations.

$$
\begin{aligned}
2x - y + z &= 1 \\
x + y - z &= -1 \\
-3x - 3y + 3z &= 3
\end{aligned}
$$

2. Explain why there is no solution to the following system of equations,

$$
\begin{aligned}
2x - 3y - 4z &= -5 \\
x - y + 3z &= -201 \\
5x - 5y + 15z &= -1004
\end{aligned}
$$

3. Solve the following systems of equations using elementary operations. Indicate whether the system is consistent or inconsistent. If consistent, how many solutions. Interpret your results geometrically.

   (a)

$$
\begin{aligned}
\frac{x}{3} - \frac{y}{4} + z &= \frac{7}{8} \\
2x + 2y - 3z &= -20 \\
x - 2y + 3z &= 2
\end{aligned}
$$

   (b)

$$
\begin{aligned}
x - y - z &= -1 \\
y - 2 &= 0 \\
x + 1 &= 5
\end{aligned}
$$

(c)

$$x - 2y + z = 3$$
$$x + y + z = 2$$
$$x - 3y + z = -6$$

(d)

$$x + y + z = 1$$
$$x - 2y + z = 0$$
$$x - y + z = 0$$

(e)

$$5x - 2y + 3z = 1$$
$$5x - 2y + 3z = -1$$
$$5x - 2y + 3z = 13$$

(f)

$$3x - 2y + z = 4$$
$$9x - 6y + 3z = 12$$
$$6x - 4y + 2z = 5$$

4. The following system of equations represents three planes that intersect in a line,

$$2x + y + z = 4$$
$$x - y + z = p$$
$$4x + qy + z = 2$$

(a) Determine p and q.

   (b) Determine an equation in parametric form for the line of intersection.

5. Consider the following system of equations,

$$
\begin{aligned}
4x + 3y + 3z &= -8 \\
2x + y + z &= -4 \\
3x - 2y + (m^2 - 6)z &= m - 4
\end{aligned}
$$

Determine the value(s) of m for which this system of equations will have,

   (a) no solution

   (b) one solution

   (c) an infinite number of solutions