Planes in $\mathbb{R}^3$

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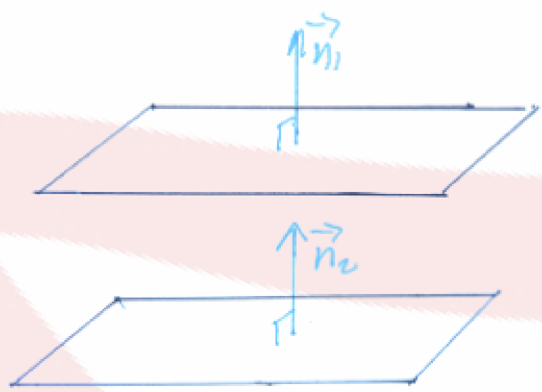
Being able to visualize lines, planes and combinations of them in 2 dimenions or 3 dimenions can go a long way to helping tackle questions. So, what do two planes in $\mathbb{R}^3$ look like geometrically? Good question.

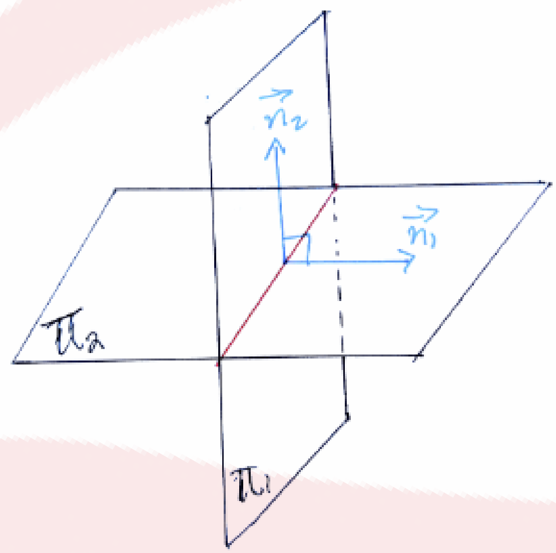
Case 1: Parallel Planes



$\vec{n_1} = k\vec{n_2}$ where $k \in \mathbb{R}$.

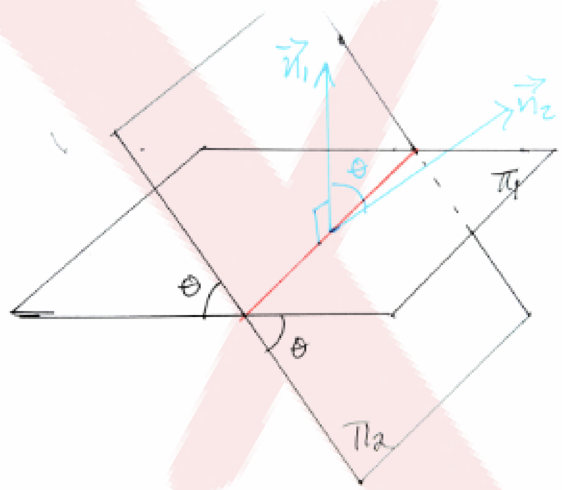
Case 2: Intersecting Planes

Perpendicular Planes



$\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0.$

Angle ≠ 90°



$$\cos\theta = \frac{\overrightarrow{n_1} \cdot \overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}$$

Recall the Cartesian equation of a plane is given by,

$$Ax + By + Cz + D = 0$$

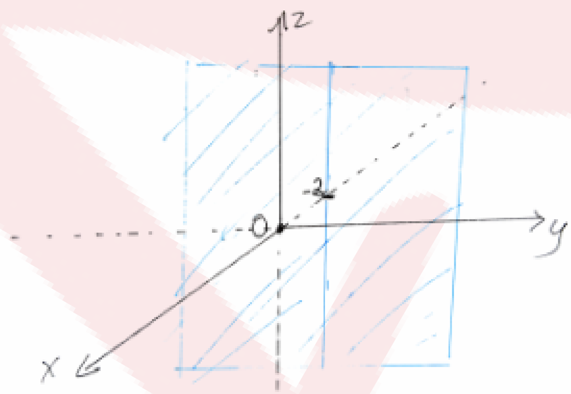
where $\overrightarrow{n} = (A, B, C)$ is normal to the plane. What does the plane look like when one or more of the scalars, A, B, C or D are equal to zero?

Case 1: Two of A, B or C are equal to zero

Let's consider the situation when $B = C = 0$. This gives the Cartesian equation,

$$Ax + D = 0.$$

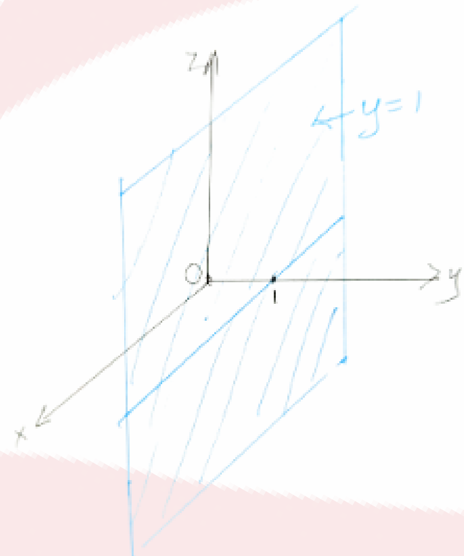
An example could be $3x + 6 = 0$ or $x = -2$. This plane passes through $x = -2$ and is parallel to the zy-plane.



Let's consider the situation when $A = C = 0$. This gives the Cartesian equation,

$$By = D = 0$$

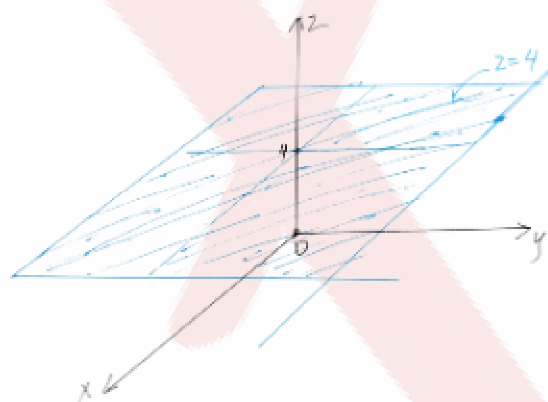
An example could be $y = 1$. This plane passes through $y = 1$ and is parallel to the xz-plane.



Let's consider the situation when $A = B = 0$. This gives the Cartesian equation,

$$Cz + D = 0$$

An example could be $z = 4$. This plane passes through $z = 4$ and is parallel to the xy-plane.

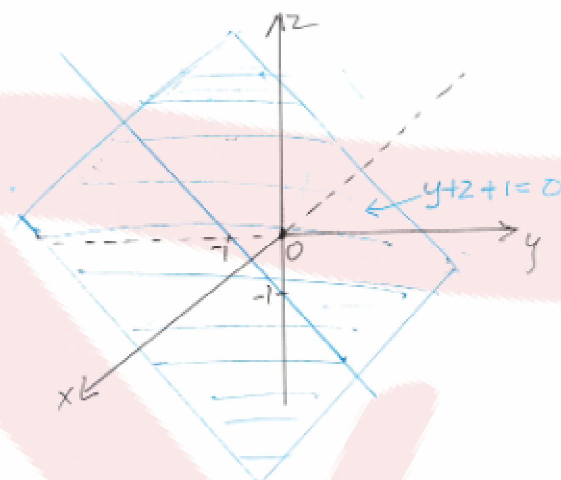


Case 2: One of A, B, C=0

Let's consider the situation when $A = 0$. This gives the Cartesian equation,

$$By + Cz + D = 0$$

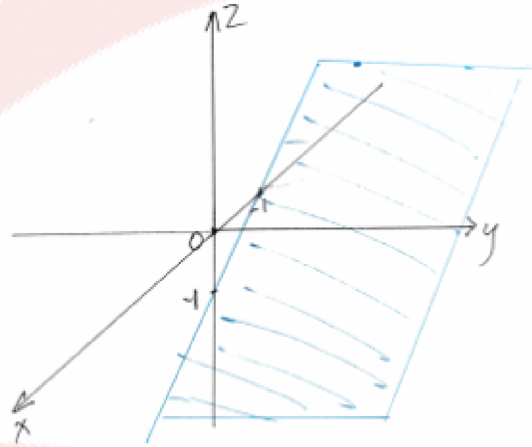
An example could be $y + z + 1 = 0$. This plane passes through the line $z = -y - 1$, parallel to the x-axis.



Let's consider the situation when $B = 0$. This gives the Cartesian equation,

$$Ax + Cz + D = 0$$

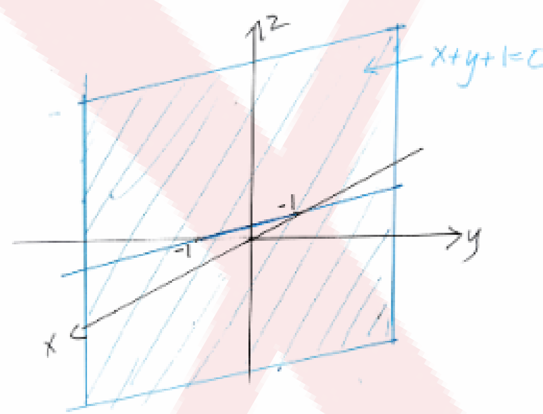
An example could be $x + z + 1 = 0$. This plane passes through the line $z = -x - 1$, parallel to the y-axis.



Let's consider the situation when $C = 0$. This gives the Cartesian equation,

$$Ax + By + D = 0$$

An example could be $x + y + 1 = 0$. This plane passes through the line $y = -x - 1$, parallel to the z-axis.

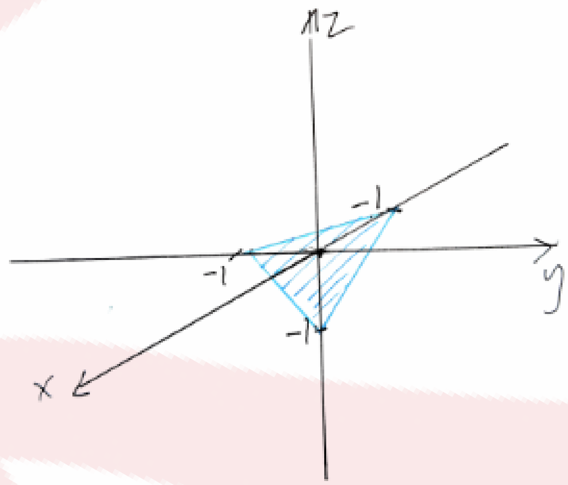


Case 3: A, B, C ≠0

Let's consider the situation when $D \neq 0$. This gives the Cartesian equation,

$$Ax + By + Cz + D = 0$$

An example could be $x + y + z + 1 = 0$. This is a plane not passing through the origin, (0, 0, 0).

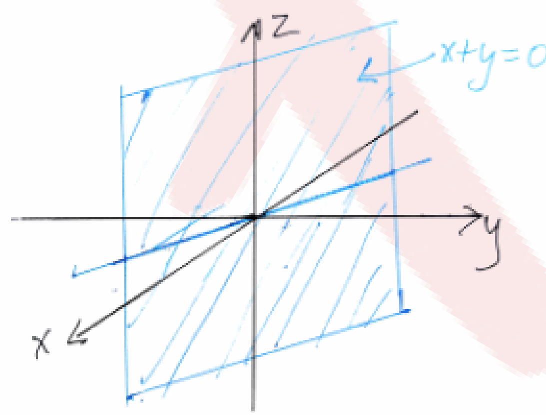


Case 4: D=0

Let's consider the situation when $D = 0$. This gives the Cartesian equation,

$$Ax + By + Cz = 0$$

An example could be $x + y + z = 0$. This is a plane passing through the origin, (0, 0, 0).



Exercises

1. Describe the following planes in words,
   (a) $x = -2$
   (b) $y = 3$
   (c) $z = 4$

2. On which of the planes, $\pi_1 : x = 5$ or $\pi_2 : y = 6$ could the point P(5, -3, -3) lie? Justify.

3. State the x, y and z intercepts for each of the following three planes and state two direction vectors for each plane.
   (a) $\pi_1 : 2x + 3y = 18$
   (b) $\pi_2 : 3x - 4y + 5z = 120$
   (c) $\pi_3 : 13y - z = 39$

4. For each of the following equation, sketch the corresponding plane,
   (a) $\pi_1 : 4x - y = 0$
   (b) $\pi_2 : 2x + y - z = 4$
   (c) $\pi_3 : z = 4$
   (d) $\pi_4 : y - z = 4$

5. For each equation below, sketch the corresponding plane,
   (a) $2x + 2y + z - 4 = 0$
   (b) $3x - 4z = 12$
   (c) $5y - 15 = 0$

