The Intersection of Planes

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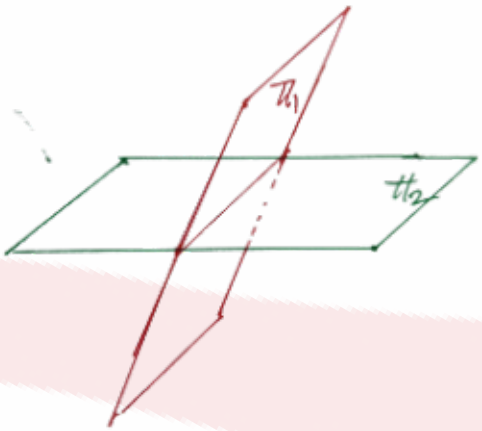
The Intersection of Two Planes

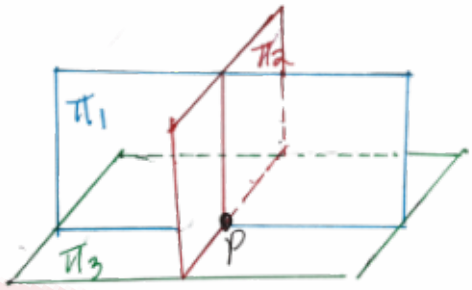
Line of intersection



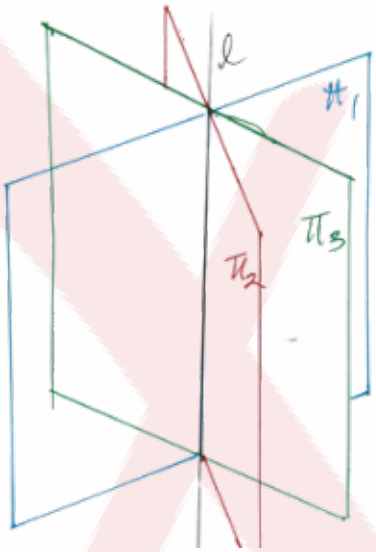
Plane of intersection

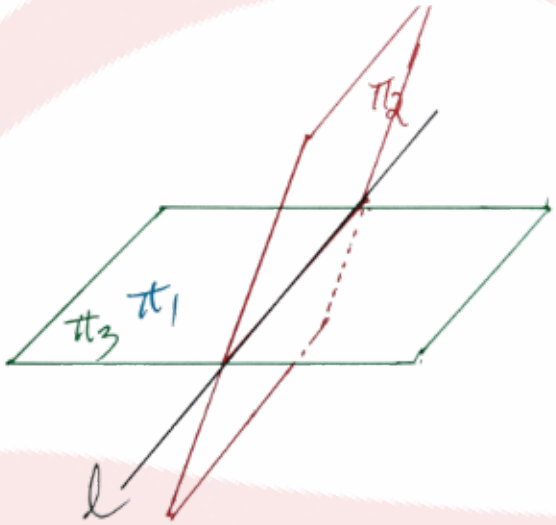
Intersection of Three Planes

Point of intersection



Line of Intersection





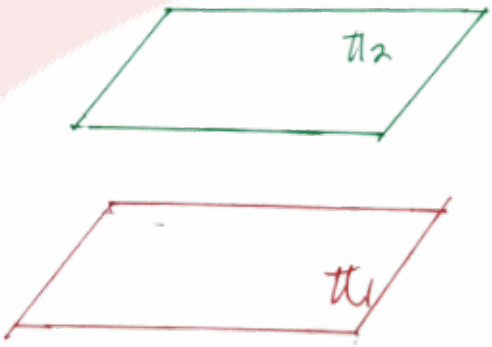
Plane of Intersection

Since the equations of lines and planes are all "linear", the above situations all result in a consistent linear system. What does the situation look like graphically when the linear system is inconsistent?

Inconsistent Systems

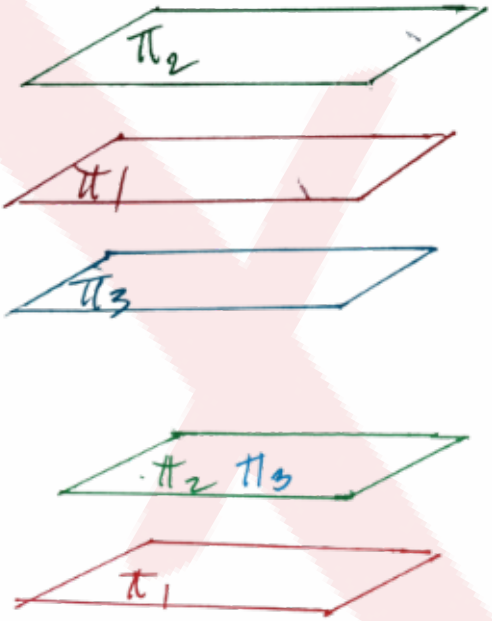
Two Planes

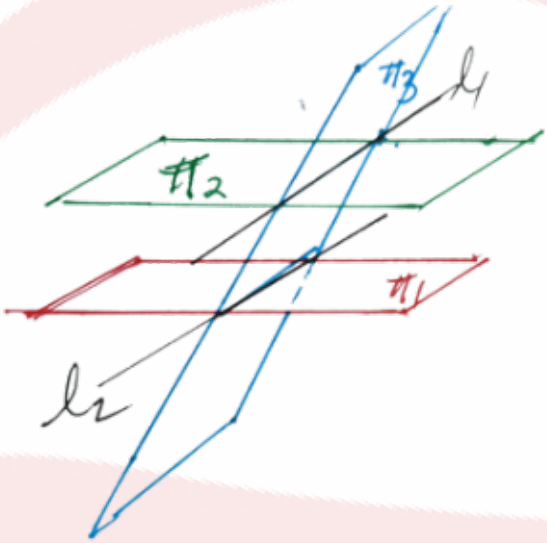
Two planes create an inconsistent system when the planes are parallel.



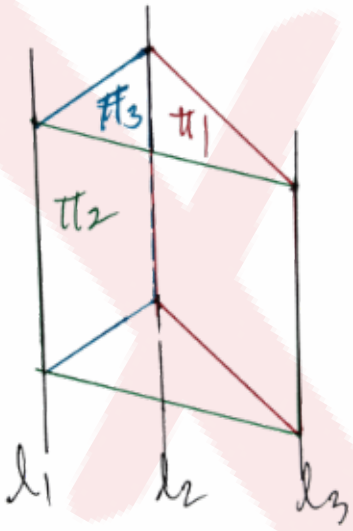
Three Planes

Three planes create an inconsistent system when at least two of the planes are parallel.





The other case when three planes create an inconsistent system is when the three planes create a *trianglar prism*.



Exercises

1. Given the following systems of questions,

   i) State whether the planes intersect. If they do, specify whether it's plane or line of intersection.

   ii) Determinethe solution of each of the systems of equations.

   (a)

$$\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 2z &= 2 \end{aligned}$$

   (b)

$$\begin{aligned} x - y + 2z &= 2 \\ x + y + 2x &= -2 \end{aligned}$$

   (c)

$$\begin{aligned} 2x - y + 2z &= 2 \\ -x + 2y + z &= 1 \end{aligned}$$

2. A system of equations is give below,

$$\begin{aligned} x + y + 2z &= 1 \\ kx + 2y + 4z &= k \end{aligned}$$

   (a) For what value of k does the system have an infinite number of solutions? Determine the solution to the system for this value of k.

   (b) Is there any value of k for which the system does not have a solution? Justify.

3. For the planes $2x - y + 2z = 0$ and $2x + y + 6z = 4$, show that their line of intersection lies on the plane with equation $5x + 3y + 16z - 11 = 0$.

4. The line of intersection of the planes $\pi_1 : 2x + y - 3z = 3$ and $\pi_2 : x - 2y + z = -1$ is a line l.
   (a) Determine parametric equations for l.
   (b) If l meets the xy-plane at point A and the z-axis at point B, determine the length of line segment AB.

