Introduction to Vectors

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2021

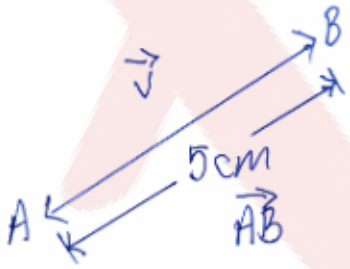
What is a vector?

A vector is a mathematical quantity that has both *magnitude* and *direction*. What exactly does this mean?

Let's consider the example of velocity. The velocity of a car is 60km/h travelling wouth east. Our car's velocity has a *magnitude* of 60km/h and a *direction* of southeast. This means that velocity is an example of a vector. Acceleration is another common vector. For example, the acceleration of the plane is 300km/h^2 northwest. Again, we have a magnitude of 300km/h^{2}and a direction of northwest. Visually, how are vectors represented? Vectors are represented by an arrow. The *length* of the arrow represents the magnitude and the position of the arrow head represents the *direction*.

Example

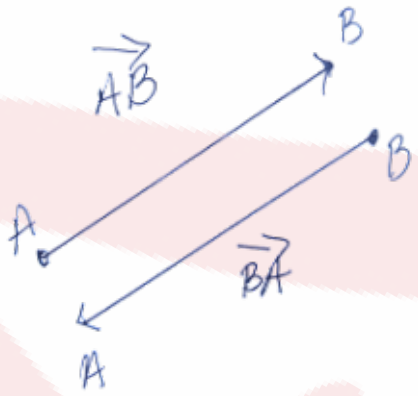
Jack walks at a velocity of 5km/h northeast. Our velocity vector $\overrightarrow{v}$ is represented by a line segment from point A to B. The length of the line segment is 5cm where 1cm is equivalent to 1km. The arrow head of the vector $\overrightarrow{v}$ is pointing in the northeast direction to represent the direction of the vector $\overrightarrow{v}$.



Notice that we have used $\overrightarrow{v}$ to denote the velocity but another way is with the line segment AB.

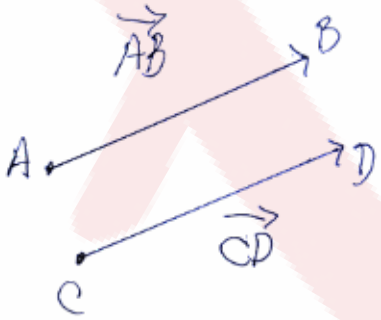
$\overrightarrow{AB}$ represents the vector for the velocity. A is the starting point and B the ending point or A is the *tail* and B the *head* of the vector. $|\overrightarrow{AB}|$ represents the *magnitude* of the vector. The magnitude is always a non-negative value.

Opposite vectors are vectors that have the same magnitude but point in opposite directions.



$|\overrightarrow{AB}| = |\overrightarrow{BA}|$ and $\overrightarrow{AB} = -\overrightarrow{BA}$.

Equal or *equivalent* vectors are two vectors that have the same magnitude and the same direction.



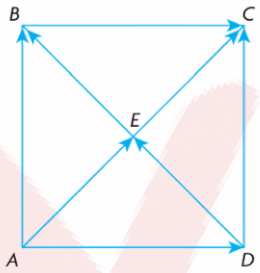
$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal vectors but notice that they are distinct, so not the same.

Note: A *scalar* is a mathematical quantity that has only magnitude. For example, speed, length and volume are all scalars.

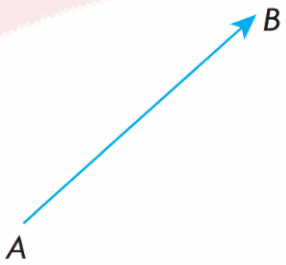
Exercises

1. For each of the following, state whether the quanity is a scalar or a vector and give a brief explanation why: height, temperature, weight, mass, area, volume, distance, displacement, speed, force, velocity, accleration

2. The square ABCD is drawn below with the diagonals intersecting at E.
   (a) State four pairs of equivalent vectors.
   (b) State four paris of opposite vectors.
   (c) State two pairs of vectors whose magnitudes are equal but whose directions are perpendicular to each other.



3. Given the vector $\overrightarrow{AB}$ shown below, draw a vector
   (a) equal to $\overrightarrow{AB}$
   (b) opposite to $\overrightarrow{AB}$
   (c) whose magnitude equalts $|\overrightarrow{AB}|$ but is not equal to $\overrightarrow{AB}$
   (d) whose magnitude is twice that of $\overrightarrow{AB}$ and in the same direction
   (e) whose magnitude is half that of $\overrightarrow{AB}$ and in the opposite direction



4. Using a scale of 1cm to represent 10km/h, draw a velocity vector to represent each of the following:
   (a) a bicyclist heading due north at 40km/h
   (b) a car heading in a southwesterly direction at 60km/h
   (c) a car travelling in northeasterly direction at 100km/h
   (d) a boy running in a northwesterly direction at 30km/h
   (e) a girl running around a circular track travelling at 15km/h heading due east

5. For each of the following vectors, describe the opposite vector.
   (a) an airplane flies due north at 400km/h
   (b) a car travels in a northeasterly direction at 70km/h
   (c) a bicyclist pedals in a northwesterly direction at 30km/h
   (d) a boat travels due west at 25km/h

6. James is running around a circular track with a circumference of 1km at a constant speed of 15km/h. His velocity vector is represented by a vector tangent to the circle. Velocity vectors are drawn at points A and C as shown. As James changes his position on the track, his velocity vector changes.
   (a) Explain why Jame's velocity can be represented by a vector tangent to the circle.

(b) What does the length of the vector represent?

(c) As he completes a lap running at a constant speed, explain why Jame's velocity is different at every point on the circle.

(d) Determine the point on the circle where James is heading due south.

(e) In running his first lap, there is a point at which James is travelling in a northeasterly direction. If he starts at a point A how long would it have taken his to get to this point?

(f) At the point he has travelled 3/8 of a lap, in what direction would James be hading? Assume he starts at point A.