Algebraic Representation of Vectors

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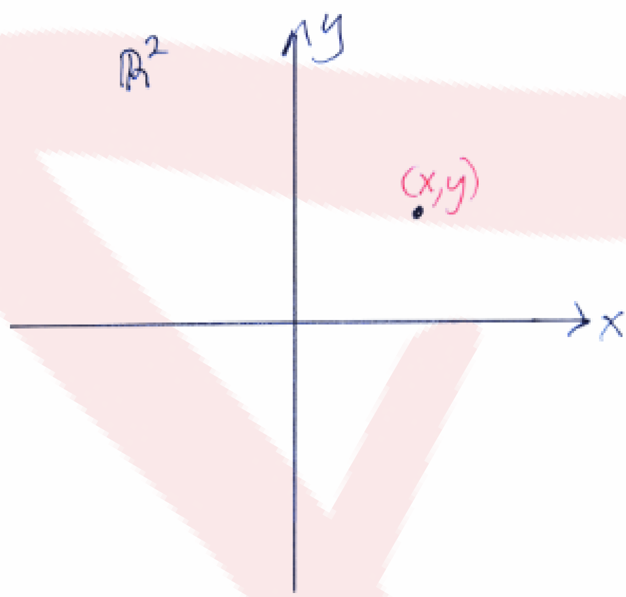
Algebraic Representation of Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$

Frist, what does $\mathbb{R}^2$ and $\mathbb{R}^3$ mean? $\mathbb{R}^2$ is a representation for 2-dimensional space or a plane. It is a set representation of a plane. That is, an element in $\mathbb{R}^2$ looks like (x, y) where $x, y \in \mathbb{R}$. This is familiar as it is the representation of a point on the Cartesian Plane. $\mathbb{R}^2$ can be thought of as the set of points representation the Cartesian plane.

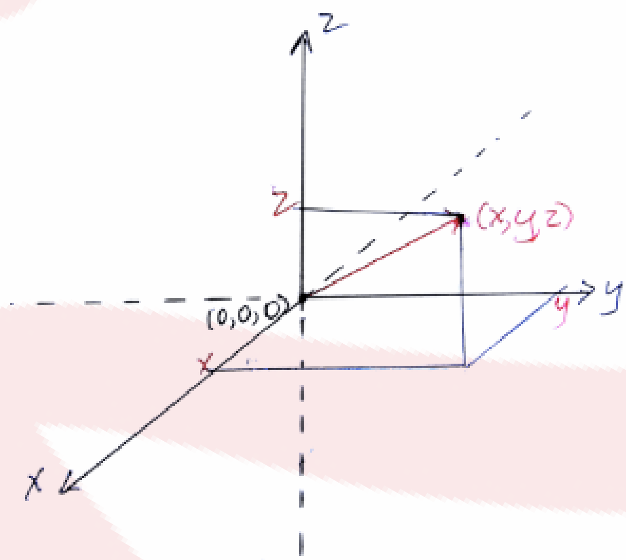


Recall the diagram above. This is familiar to use and represents the Cartesian plane and the set of points in this plane which can be denoted by $\mathbb{R}^2 = \{(x, y)|x, y \in \mathbb{R}\}$.

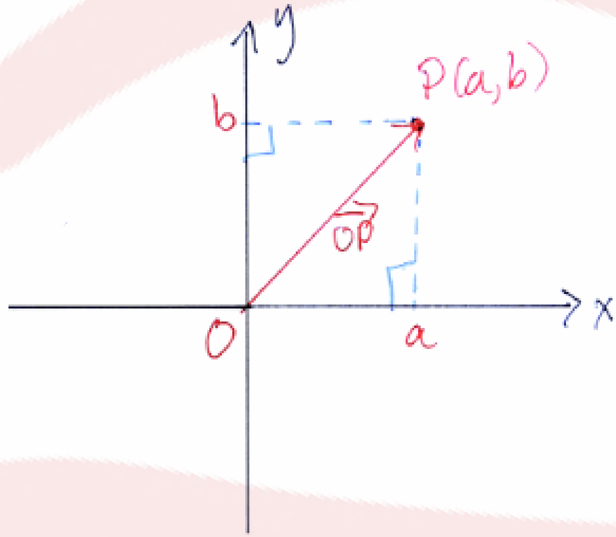
What is $\mathbb{R}^3$? If we extend the definition of $\mathbb{R}^2$ to $\mathbb{R}^3$ we have,

$$\mathbb{R}^3 = \{(x, y, z)|x, y, z \in \mathbb{R}\}$$

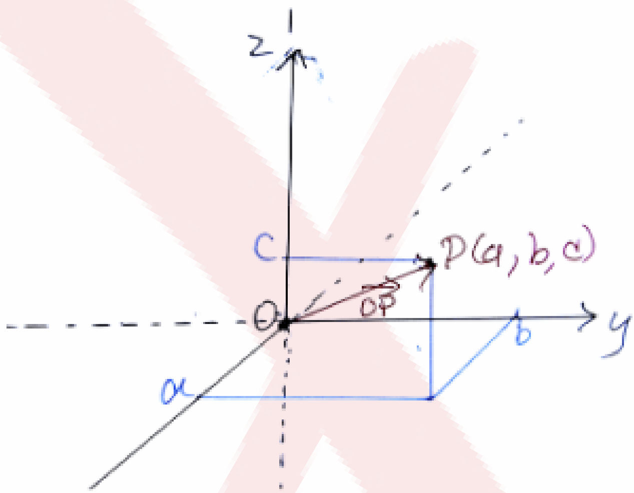
Now, what visually does $\mathbb{R}^3$ represent? Good question. $\mathbb{R}^3$ actually represents 3-dimensional space. The world that we live in. Graphically we now have three axes, x, y, z; a point in this graph has three coordinates (x, y, z).



How are vectors represented in $\mathbb{R}^2$ and $\mathbb{R}^3$? Another good question. We are going to define tsomething called a *position vectir*. A *position vector* in $\mathbb{R}^2$ or $\mathbb{R}^3$ is a vector with *head* at the point P in either $\mathbb{R}^2$ or $\mathbb{R}^3$ and *tail* at the origin. more specifically, in $\mathbb{R}^2$:
If our point P is (a, b) and our origin is $O(0, 0)$ then our position vector for P is denoted by $\overrightarrow{OP}$ and graphically looks like,



In $\mathbb{R}^3$: If our point P is (a, b, c) and our origin is $O(0, 0, 0)$ then the position vector for P is denoted by $\overrightarrow{OP}$ and graphically looks like,



How do we write the position vector algebraically? Let's consider an example.

Example

For the following points in either $\mathbb{R}^2$ or $\mathbb{R}^3$ write the position vector algebraically,

(a) P(2,3)

(b) P(-1, 1, 0)

(c) P(2, -3, 1)

Solution

(a) P(2,3) is in $\mathbb{R}^2$. The position vector is $\overrightarrow{OP}$ and given by ,

$$\begin{aligned} P - O &= (2,3) - (0,0) \\ &= (2-0, 3-0) \\ &= (2,3) \\ &= \overrightarrow{OP} \end{aligned}$$

Therefore, $\overrightarrow{OP} = (2,3)$

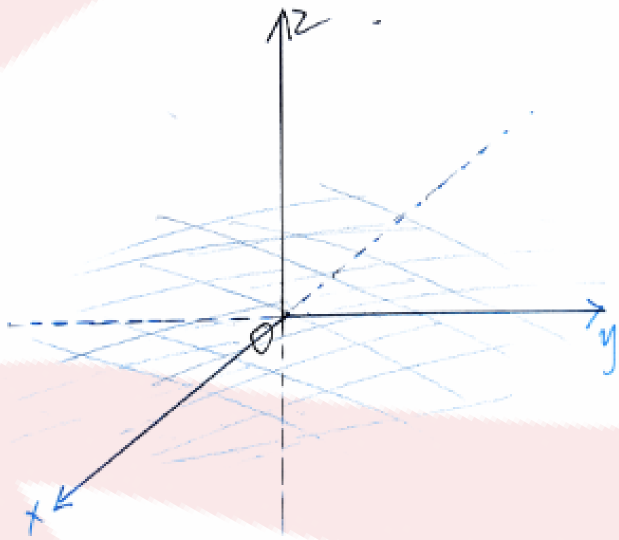
(b) P(-1, 1, 0) is in $\mathbb{R}^3$. The position vector $\overrightarrow{OP}$ is,

$$P - O = (-1, 1, 0 - (0,0,0) = (-1,1,0)$$

Therefore, $\overrightarrow{OP} = (-1,1,0)$.

(c) P(2, -3, 1) $\in \mathbb{R}^3$. Therefore, $\overrightarrow{OP} = (2,-3,1)$.

Representing Planes and lines in $\mathbb{R}^3$



The $x - y$ plane in $\mathbb{R}^3$ occurs when $z = 0$ for any values of x and y. Or,

$$\{(x, y, 0)|x, y \in \mathbb{R}\}$$

Suppose we want to represent the x axis in $\mathbb{R}^3$. The x-axis occurs when $z = 0$ and $y = 0$ in $\mathbb{R}^3$ or $\{(x, 0, 0)|x \in \mathbb{R}\}$.

Using these above ideas we can represent some planes and lines in $\mathbb{R}^3$. let's consider an example.

Example

(a) The plane through $y = -3$ parallel to the xy plane.

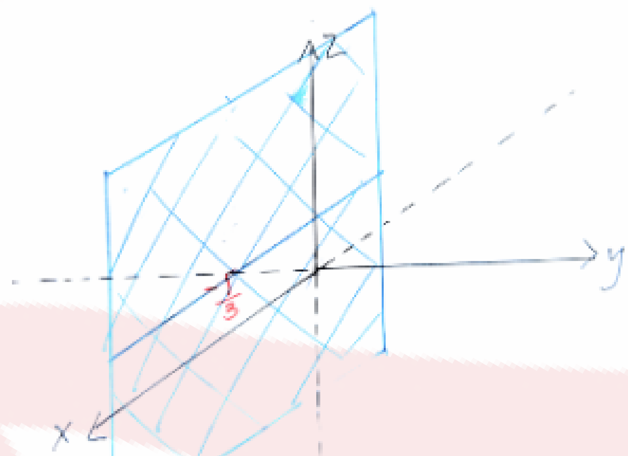
(b) The line going through $x = 2, z = 1$.

(c) The plane $x = 4$.

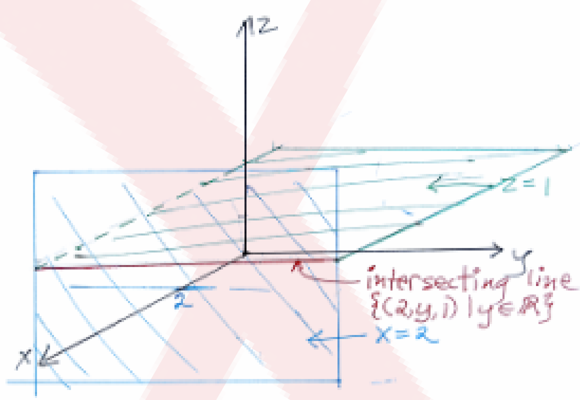
(d) The line that is the intersection of $y = -1, x = 3$.

Solution

(a) Let's start with a diagram.



$\{(x, -3, z)|x, z \in \mathbb{R}\}$



(b)

$\{(2, y, 1)|y \in \mathbb{R}\}$

(c) $\{(4, y, z)|y, z \in \mathbb{R}\}$

(d) $\{(3, -1, z)|z \in \mathbb{R}\}$

Exercises

1. Suppose that $\overrightarrow{OP} = (a, -3, c)$ and $\overrightarrow{OP} = (-4, b, -8)$. What are the correspondign values for a, b and c?

2. (a) The points $A(5, b, c)$ and $B(a, -3, 8)$ are located at the same point in $\mathbb{R}^3$. What are the values of a, b and c?

   (b) Write the vector corresponding to $\overrightarrow{OA}$.

3. Locate the points $A(4, -4, -2), B(-4, 4, 2)$ and $C(4, 4, -2)$ using coordinate axes that you construct yourself. Draw the corresponding rectangular box (prism) for each, and label the coordinate of its vertices.

4. (a) On what axis is $A(0, -1, 0)$ located? name three other points on this axis.

   (b) Name the vector $\overrightarrow{OA}$ associated with point A.

5. Draw a set of x, y and z-axes and plot the following points:

   (a) A(1, 0, 0)

   (b) B(0, -2, 0)

   (c) C(0, 0, -3)

   (d) D(2, 3, 0)

   (e) E(2, 0, 3)

   (f) F(0, 2, 3)

6. (a) Draw a set of x, y and z-axes and plot the following points A(3, 2, -4), B(1, 1, -4) and C(0, 1, -4).

   (b) Determine the equation of the plan containing the points A, B and C.

7. P(2, a-c, a) and Q(2, 6, 11) represent the same point in $\mathbb{R}^3$.

(a) What are the values of a and c?

(b) Does $|\overrightarrow{OP}| = |\overrightarrow{OQ}|$? Explain.

8. Each of the points P(x, y,0), Q(x, 0, z) and R(0, y, z) represent general points on three different planes. Name the three planes to which each corresponds.

9. (a) What is the equation of the plane that contains the points M(1, 0, 3), N(4, 0, 6) and P(7, 0, 9)? Explain your answer.

   (b) Exaplin why the plane that contains the points M, N and P also contains the vectors $\overrightarrow{OM}, \overrightarrow{ON}$ and $\overrightarrow{OP}$.