Implicit Differentiation

RaiseMyMarks.com

2021

Implicit Differentiation

Sometimes we're asked to find the derivative of a function but the function isn't written explicitly in terms of a single independent variable, x. For example,

$$3x^2 + 2y = 6 \tag{1}$$

One approach is to rearrange the equation so that the dependent variable is expressed as an expression of the indepedent variable. This is possible for the equation in (1) but is not quite as straight forward for other equations such the example below,

$$x^3 + y^3 = 3xy \tag{2}$$

In the situation presented in (2) we use something called *implicit differentiation* to find the derivative of y. How do we perform *implicit differentiation*? First we keep in mind that y is a function of x.

Steps for implicit differentiation

1. Take the derivative of each term in the equation.
2. When taking the derivative of a dependent variable, y say, keep in mind that the *chain rule* must be applied. For example,

$$\frac{d(y^3)}{dx} = 3y^2\frac{dy}{dx}$$

3. Finally, solve for $\frac{dy}{dx}$.

Let's consider an example.

Example

Use implicit differentiation to find $\frac{dy}{dx}$.

$$xy + y^2 = 1 \tag{3}$$

Solution: Let's take the deriviative of both sides of (3).

$$\begin{aligned}
\frac{d(xy+y^2)}{dx} &= \frac{d(1)}{dx} \\
y + x\frac{dy}{dx} + 2y\frac{dy}{x} &= 0 \\
y + (x+2y)\frac{dy}{dx} &= 0 \\
(x+2y)\frac{dy}{dx} &= -y \\
\frac{dy}{dx} &= -\frac{y}{x+2y}
\end{aligned}$$

Therefore,

$$\frac{dy}{dx} = -\frac{y}{x+2y}$$

Exercises

Use the chain rule to differentiate the following functions.

a) $y = x^{3/4}$

f) $(y-2)^2 = x+1$

b) $y^{-2} = x^2 + x$

g) $5x^2 + 5y^2 = 12$

c) $y^3 = x^2 + x - 2$

h) $16x^2 + 9y^2 = 144$

d) $10x^2 + 2y^2 = 20$

i) $x^2 - 4y^2 = -16$

e) $9x^2 - 4y^2 = -36$

j) $x^{1/3} + y^{1/3} = 1$

k) $(x+y)^2-(x-y)^2=x^4+y^4$ l) $x^2y^2=x^2+y^2$