Higher Order Derivatives

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So far we have considered taking a derivative of a function, implicitly or explicitly, once only. This is the first derivative we have been considering. When we take the derivative of the first derivative, we have a second derivative; the derivative of the second derivative is the third derivative; and so on. Notation-wise we have the following:

Function	$f(x) = y$
1st derivative	$f'(x) = \frac{dy}{dx}$
2nd derivative	$f''(x) = \frac{d^2y}{dx^2}$
3rd derivative	$f'''(x) = \frac{d^3y}{dx^3}$

Let's consider an example.

Example

Find the second derivative of $f(x) = \frac{x}{1+x}$.

Solution: We need to use the *quotient rule.*

$$f'(x) = \frac{1+x-x}{(1+x)^2} = \frac{1}{(1+x)^2}$$
$$f''(x) = (1+x)^{-2}$$

Now we can use the power rule.

$$f''(x) = -2(1+x)^{-3}$$

Let's fid the third derivative of the function above.

$$f'''(x) = (-2)(-3)(1+x)^{-4} = 6(1+x)^{-4}$$

Exercises

Find the second derivative of the following functions,

a) $f(x) = 8x^4 + \frac{1}{3}x^2 + 9x - 5$

b) $f(x) = 2x^5 - 7x^3 + 9x^2$

c) $f(x) = \sin 5x$

d) $f(x) = \tan 3x$

e) $f(x) = \ln(9x + 2)$

f) $f(x) = e^{x^2+1}$

g) $f(x) = \frac{1+x^2}{x^2+2x-1}$

h) $Q(v) = \frac{2}{(6+2v=+v^2)^4}$

i) $y = e^{-5x} + 8\ln(2x^4)$

j) $y = 4\sqrt[5]{x^3}$

k) $y = \frac{1}{8x^2} - \sqrt{x}$