Maximum and Minimum Values

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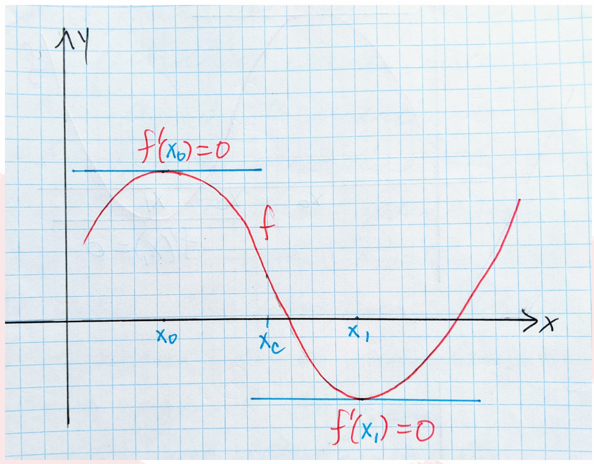
Maximum and Minimum Values

We know that the derivative of a function f at a particular point $x = a$ is the slope of the tangent othe function at the point $P(a, f(a))$. When we are at a maximum or minimum value of a function, what is the value of the derivative? Let's take a look.

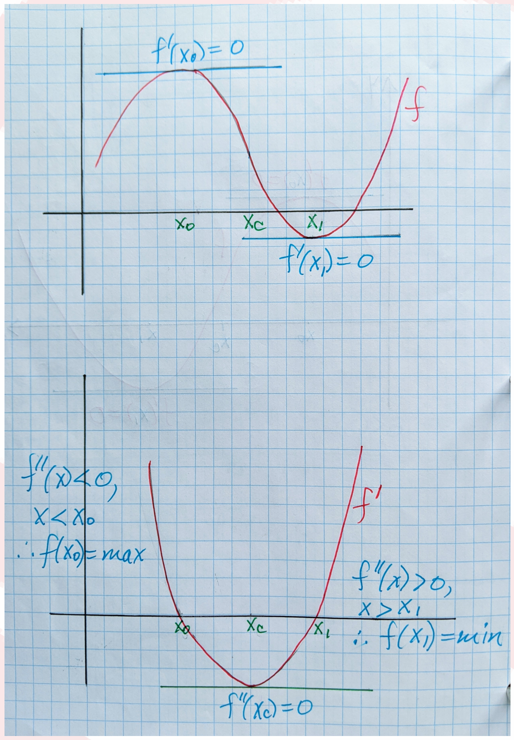


Therefore,

$$
\begin{aligned}
f''(x_0) < 0 \quad &\Longrightarrow \quad f(x_0) = maximum \\
f''(x_1) > 0 \quad &\Longrightarrow \quad f(x_1) = minimum
\end{aligned}
$$

Notice that at the points where the function is a maximum $x = x_0$ and a minimum $x = x_1$, the derivative of the function at these points is 0, $f'(x_0) = 0$ and $f'(x_1) = 0$, because the tangents are horizontal and so have slope equal to zero. How do we determine where a function has a maximum or minimum?

We solve $f'(x) = 0$ for x.



Therefore we have,

$$f'(x_0) = 0, \;\; f''(x_0) < 0 \quad \Longrightarrow \quad f(x_0) = maximum$$
$$f'(x_1) = 0, \;\; f''(x_1) > 0 \quad \Longrightarrow \quad f(x_1) = minimum$$

Procedure for finding the maximum and minimums of a function

1. Solve $f'(x) = 0$ for x. Let $x = x_0$ be such that $f'(x_0) = 0$.
2. Calculuate $f''(x_0)$.
3. If $f''(x_0) < 0$ then $f(x_0)$ is a maximum. If $f''(x_0) > 0$ then $f(x_0)$ is a minimum.
4. Solve $f''(x) = 0$. Let $x = x_c$ be such that $f''(x_c) = 0$.

5. $x = x_c$ is called the *point of inflection* and is the point where the " concavity" of the function changes.

Exercises

1. Find the maximum value of each function on the given interval.

a) $f(x) = x^2 - 4x + 3,\ 0 \leq x \leq 3$

b) $f(x) = x^3 - 3x^2,\ -1 \leq x \leq 3$

c) $f(x) = x^3 - 3x^2,\ -2 \leq x \leq 1$

d) $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x,\ 0 \leq x \leq 4$

e) $f(x) = x + \frac{4}{x},\ 1 \leq x \leq 10$

f) $f(x) = 4\sqrt{x} - x,\ 2 \leq x \leq 9$

g) $f(x) = 3x^4 - 4x^3 - 36x^2 + 20,\ -3 \leq x \leq 4$

h) $f(x) = \frac{4x}{x^2+1},\ -2 \leq x \leq 4$

2. Find the minimum for each function in # 1 on the given interval.