Higher Order Derivatives

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So far we have considered taking a derivative of a function, implicitly or explicitly, once only. This is the first derivative we have been considering. When we take the derivative of the first derivative, we have a second derivative; the derivative of the second derivative is the third derivative; and so on. Notation-wise we have the following:

Function	$f(x) = y$
1st derivative	$f'(x) = \frac{dy}{dx}$
2nd derivative	$f''(x) = \frac{d^2y}{dx^2}$
3rd derivative	$f'''(x) = \frac{d^3y}{dx^3}$

Let's consider an example.

Example

Find the second derivative of $f(x) = \frac{x}{1+x}$.

Solution: We need to use the *quotient rule*.

$$\begin{aligned} f'(x) &= \frac{1+x-x}{(1+x)^2} = \frac{1}{(1+x)^2} \\ f''(x) &= (1+x)^{-2} \end{aligned}$$

Now we can use the power rule.

$$f''(x) = -2(1+x)^{-3}$$

Let's fid the third derivative of the function above.

$$f'''(x) = (-2)(-3)(1+x)^{-4} = 6(1+x)^{-4}$$

Exercises

Find the following derivatives:

a) $x^2 + y^2 = 36$ First derivative.

b) $15y^2 = 2x^3$ Secod derivative.

c) $3xy^2 + y^3 = 8$ Second derivative.

d) $9x^2 - 16y^2 = -144$ First derivative

e) $3x^2 + 4xy^3 = 9$ Third derivative.

f) $x^3y^3 = 144$ Second derivative

g) $x = y + y^5$ Third derivative

h) $xy^3 - x^3y = 2$ First derivative

i) $\sqrt{x} + \sqrt{y} = 5$ Third derivative.