

Concavity and Points of Inflections

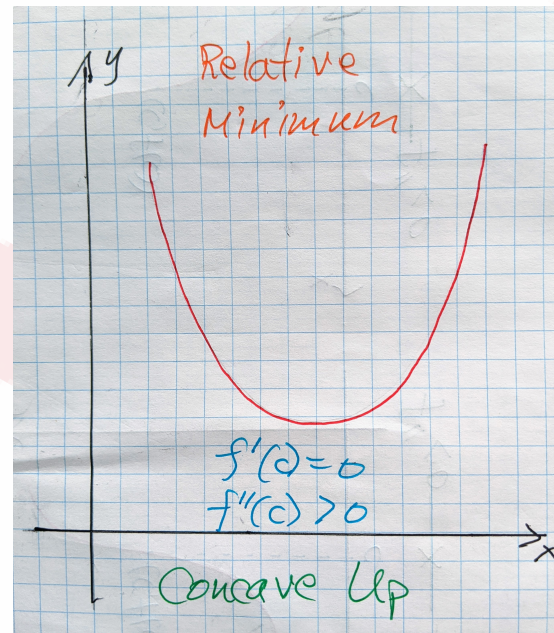
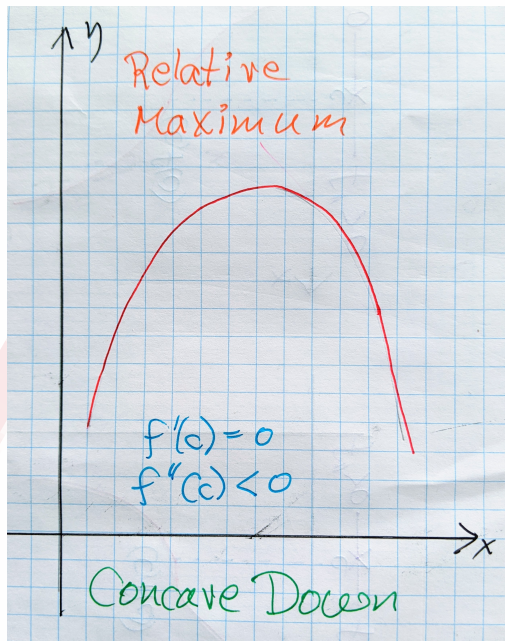
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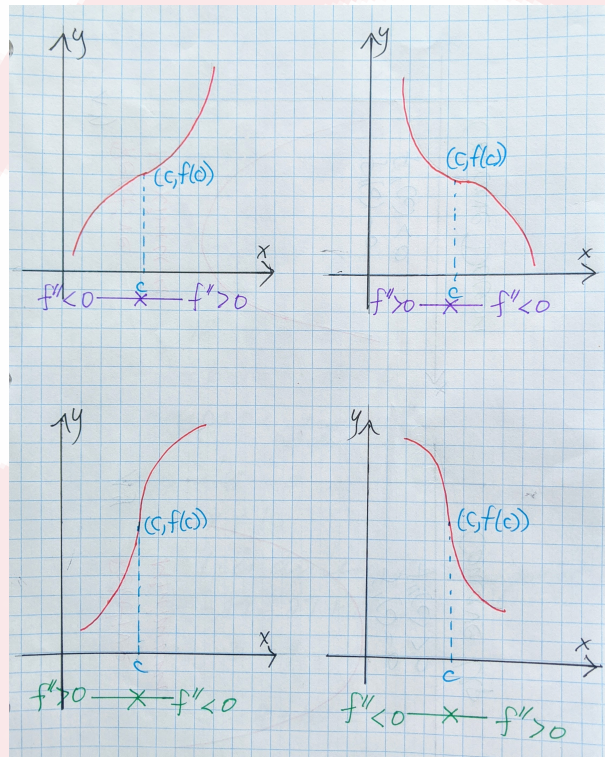
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## Concavity and points of inflection

At a critical point the function is a *relative maximum* or a *relative minimum*. The graph is said to be *concave down* at a *relative maximum* and *concave up* at a *relative minimum*.



A *point of inflection* is a critical point  $(c, f(c))$  where  $f''(c) = 0$ . It is a point where the concavity of the function changes.



Let's consider a few examples.

### Example

Determine points of inflection for

$$f(x) = \frac{1}{x^2 + 3}$$

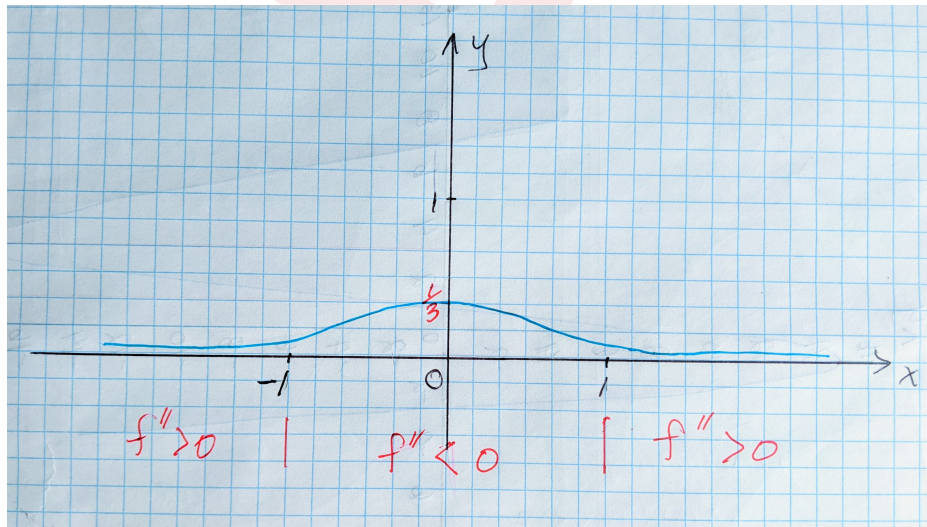
**Solution:**

$$\begin{aligned}
 f'(x) &= -(x^2 + 3)^{-2}(2x) \\
 &= \frac{-2x}{(x^2 + 3)^2} \\
 f''(x) &= \frac{-2(x^2 + 3) + (-2x)(2)(x^2 + 3)(2x)}{(x^2 + 3)^4} \\
 &= \frac{-2(x^2 + 3) + 8x^2}{(x^2 + 3)^3} \\
 &= \frac{6x^2 - 6}{(x^2 + 3)^3}
 \end{aligned}$$

Therefore,  $x = +1$  or  $-1$  are points of inflection;  $f'(x) = 0$ ,  $x = 0$  is a critical point. We have,

$$f''(0) = -\frac{6}{3^3} = -\frac{6}{27} = -\frac{2}{9} < 0$$

Therefore, at  $x = 0$  is a relative maximum or  $f$  is a concave down at  $x = 0$ .



**Example**

Graph

$$f(x) = \frac{x - 4}{x^2 - x - 2}$$

**Soution:**

$$f(x) = \frac{x - 4}{x^2 - x - 2} = \frac{x - 4}{(x - 2)(x + 1)}$$

Therefore,  $x = 2, -1$  are discontinuities.

	-1	2	4
$x - 4$	-	-	+
$x - 2$	-	-	+
$x + 1$	-	+	+
$f(x)$	-	+	+

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

Therefore,  $x = 2, x = -1$  are vertical asymptotes.

$$\lim_{x \rightarrow \infty} \frac{x - 4}{x^2 - x - 2} = \lim_{x \rightarrow \infty} \frac{x(1 - 4/x)}{x^2(1 - 1/x - 2/x^2)} = 0$$

Therefore,  $y = 0$  is a horiztonal asymptote.

**Exercises**

1. Find the second derivative for the following at the given point.

a)  $f(x) = 2x^3 - 10x + 3, x = 2$     b)  $g(x) = x^2 - \frac{1}{x}, x = -1$

c)  $s = e^t \ln t, t = 1$

2. For each function in #1 determine the points of inflection.

3. For the functions in #1 determine whether the function lies above or below the function at the given point.