# Derivative of Exponential Function


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# Derivatives of the exponential function

## Derivative of $e^x$

Let's use the definition of the derivate to find the derivative of $f(x) = b^x$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{b^{x+h}-b^x}{h} \\ &= \lim_{h\to 0} \frac{b^x(b^h-1)}{h} \\ &= b^x \lim_{h\to 0} \frac{b^h-1}{h} \\ f'(x) &= b^x f'(0) \end{aligned}$$

Therefore,

$$\boxed{f'(x) = b^x f'(0)}$$

Let's consider the value of $b = e$ where $e = 2.718281\ldots$.

$$\boxed{For f(x) = e^x,\ f'(x) = e^x.}$$

In general,

$$\boxed{f(x) = e^{g(x)} \Rightarrow f'(x) = g'(x)e^{g(x)}}$$

  

## Exercises

Differentiate the following.

a) $y = e^{3x}$

b) $s = e^{3t-5}$

c) $y = e^{\sqrt{x}}$

d) $y = 2e^{x^2}$

e) $f(x) = \frac{e^{-x^3}}{x}$

f) $f(x) = \sqrt{x}e^x$

g) $h(t) = e^{t^2} + 3e^{-t}$

h) $g(t) = \frac{e^{2t}}{1+e^{2t}}$

i) $f(x) = \frac{1}{3}(e^{3x} + e^{-3x})$

j) $f(x) = e^{-\frac{1}{x+1}}$

k) $y = \frac{5}{2}(e^{x/5} + e^{-x/5})$

m) $x^2 e^y = 1$

l) $y - e^{xy} = 0$