

Rules of Differentiation

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## Rules of differentiation

There are a number of rules when taking the derivative of a function.  
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### Constant Function Rule

If  $f(x) = K$  then  $f'(x) = 0$ .

### The Power Rule

If  $f(x) = x^n$  then  $f'(x) = nx^{n-1}$  where  $n$  is a real number,  $n \in \mathbb{R}$ .

### Constant Multiple Rule

If  $f(x) = Kg(x)$  where  $K$  is a constant then  $f'(x) = Kg'(x)$ .

### The Sum Rule

If  $f(x)$  and  $g(x)$  are differentiable functions and  $F(x) = f(x) + g(x)$  then  $F'(x) = f'(x) + g'(x)$ .

### Difference Rule

If  $F(x) = f(x) - g(x)$  then  $F'(x) = f'(x) - g'(x)$

### The Product Rule

If  $F(x) = f(x)g(x)$  then  $F'(x) = f'(x)g(x) + f(x)g'(x)$

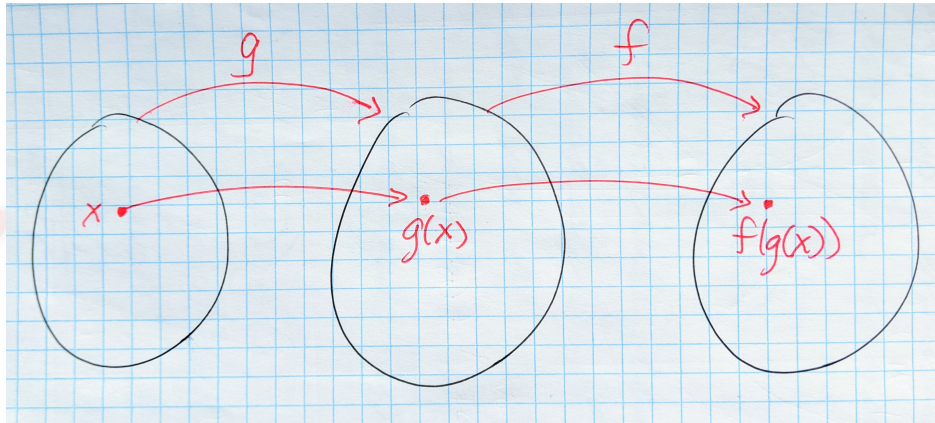
### Power Rule

If  $f(x) = [g(x)]^n$  then  $f'(x) = n[g(x)]^{n-1}g'(x)$ , where  $n \in \mathbb{Z}$ ,  $n$  is an integer.

## The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or



The domain of one function is the range of the other function. Given two functions  $f$  and  $g$  the composite function  $f \circ g$  is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

## The Chain Rule

If  $f$  and  $g$  are differentiable functions and  $F(x) = f \circ g(x)$  then

$$F'(x) = f'(g(x))g'(x)$$

Using Leibniz Notation, If  $y = f(u)$ ,  $u = g(x)$  then,

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

**Note:** The power function rule is a special case of the chain rule where  $f(x) = x^n$  and given some function  $g(x)$ , the derivative of  $F(x) = f \circ g(x) = [g(x)]^n$  is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

**Exercises**

Use the rules of differentiation to differentiate the following functions.

a)  $f(x) = (x^2 + 3x + 5)^6$

f)  $y = 3x(x - 4)(x + 3)$

b)  $f(x) = 2x(x+1)^3(x^2+2x+1)^2$

g)  $y = (2x + 1)^5(3x + 2)^4$

h)  $y = (4x^2 + 2x)(3 - 2x - 5x^2)$

c)  $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

i)  $y = (1 - 2x)(1 + 2x)$

d)  $y = 2(x - 29)(x + 1)$

j)  $y = (1 - x^2)^4(2x + 6)^3$

e)  $y = (x^3 - 5x + 2)(3x^2 - 2x)$

k)  $y = (5x + 1)^3(2x + 6)^3$

n)  $h(x) = (5x^7 + 1)(x^2 - 2x)$

l)  $y = (5x + 1)^3(x - 4)$

o)  $h(x) = x^2(2x - 1)$

m)  $s(t) = (t^2 + 1)(3 - 2t^2)$