Rules of Differentiation

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Rules of differentiation

There are a number of rules when taking the derivative of a function.
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Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where n is a real number, $n \in \mathbb{R}$.

Constant Multiple Rule

If $f(x) = Kg(x)$ where K is a constant then $f'(x) = Kg'(x)$.

The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

Difference Rule

$$\text{If } F(x) = f(x) - g(x) \text{ then } F'(x) = f'(x) - g'(x)$$

The Product Rule

$$\text{If } F(x) = f(x)g(x) \text{ then } F'(x) = f'(x)g(x) + f(x)g'(x)$$

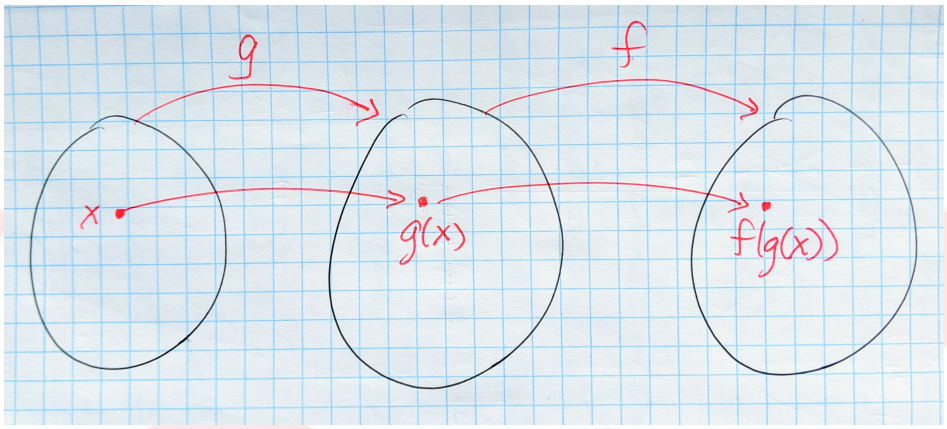
Power Rule

If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1}g'(x)$, where $n \in \mathbb{Z}$, n is an integer.

The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's' look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or



The domain of one function is the range of the other function. Given two functions f and g the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

The Chain Rule

If f and g are differentiable functions and $F(x) = f \circ g(x)$ then

$$\boxed{F'(x) = f'(g(x))g'(x)}$$

Using Leibniz Notation, If $y = f(u),\ u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

Note: The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

Exercises

Use the rules of differentiation to differentiate the following functions.

a) $f(x) = 3x^{2/3} - 6x^{1/3} + x^{-1/3}$

b) $f(x) = \sqrt{x} + 6\sqrt{x^3} + \sqrt{2}$

c) $f(x) = \frac{1+\sqrt{x}}{x}$

d) $f(x) = 9x^{-2} + 3\sqrt{x}$

e) $f(x) = 20x^5 + 3\sqrt[3]{x} + 17$

f) $f(x) = \frac{1+\sqrt{x}}{x}$

g) $s = -2t^2 + 7t$

h) $s = (t-3)^2$

i) $y = \frac{1}{x}\left(x^2 + \frac{1}{x}\right)$

j) $y = \frac{\sqrt{x}-2}{\sqrt{x}}$

k) $y = \frac{1}{x}\left(x^2 + \frac{1}{x}\right)$

m) $y = -x^2 + 3x + 4$

l) $y = \frac{3}{x^2} - \frac{4}{x^3}$

