Tangents and Areas

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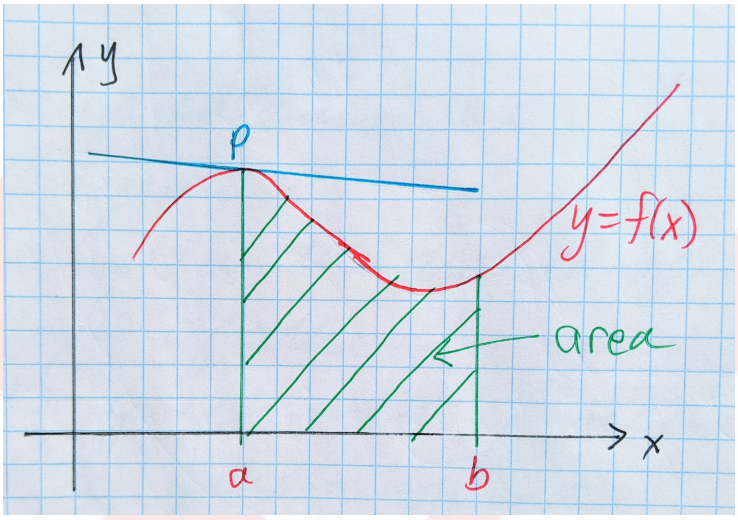
2021

Tangents and areas

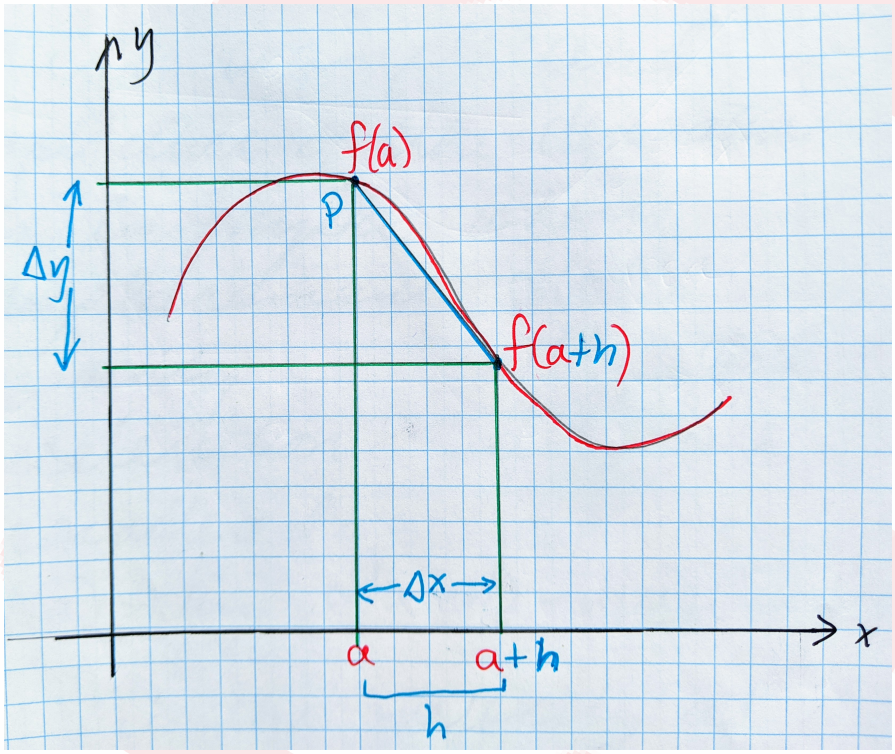
Good question. I immediately think of derivatives and integrals. But, why do we study derivatives and integrals? There are two main geometric problems that are considered. First, let's consider the graph of the function $y = f(x)$.



The problems are,

1. **Tangents:** What is the value of the slope of the tangent to the graph of a function at a given point P?
2. **Areas:** What is the area nder the graph of a function $y = f(x)$ between $x = a$ and $x = b$?

This is where the derivative comes in or was developed, for the first problem, and where the integrals come in for the area under a curve, or second problem.



The slope of the tangent to the graph $y = f(x)$ at the point $P(a, f(a))$ is

$$\begin{aligned} m &= \lim_{\Delta \to 0} \frac{\Delta y}{\Delta x} \\ &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \end{aligned}$$

This value m is called the *the derivative of f at the point $x = a$* and may be denoted by $f'(a) = m$.

Distance, velocity, acceleration

A nice example and application of the derivative is in physics where,

$$\begin{aligned} \text{displacement} &= d(t) \\ \text{veolocity} &= v(t) \\ \text{acceleration} &= a(t) \end{aligned}$$

are considered.

$$v(a) = \lim_{h \to 0} \frac{d(a+h) - d(a)}{h} = d'(a)$$

is the velocity at time $t = a$.

$$a(a) = \lim_{h \to 0} \frac{v(a+h) - v(a)}{h} = v'(a)$$

is the acceleration at time $t = a$. Notice, that since velocity is the derivative of displacement, and acceleration is the derivative of velocity. This means, acceleration is the 2^{nd} derivative of displacement of

$$a'(a) = d''(a)$$

Exercises

1. Find the slope of the tangent to each curve at the given point.

   a) $y = 3x^3$ at $x = 1$

   b) $y = \sqrt{16 - x}$ at $x = 5$

   c) $y = \frac{4+x}{x-2}$ at $x = 8$

   d) $y = \sqrt{x - 7}$ at $x = 16$

   e) $y = \sqrt{25 - x^2}$ at $(3, 4)$

   f) $y = \frac{8}{\sqrt{x+11}}$ at $x = 5$

2. A construction worker drops a bolt while working on a high-rise building $320m$ above the ground. After t seconds, the bolt has fallen s metres where $s(t) = 320 - 5t^2$, $0 \leq t \leq 8$.

   a) Find the average veloctiy during the $1^{st}, 3^{rd}$ and 8^{th} seconds.

   b) Find the average velocity for the interval $3 \leq t \leq 8$.

   c) Find the velocity at $t = 2$.

