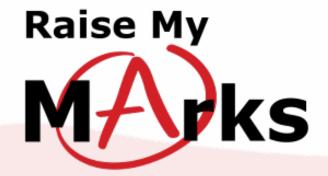
Derivatives



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Derivative

Derivative

The derivative of a function f at a point x = s may be though tof as the slope of the tangent to the cuve at the point x = a. Or the rate of change of the function f at the point x = a. How is the derivative defined in mathematical notation?

Definition of the derivative

The derivative of a function f at the point x = a is given by,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that the limit exits. Or,

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

or the derivative of f w.r.t. x is given by,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

Examples

Let's consider some examples. Find the derivative of $f(x) = x^2$ at x = -3.

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Solution:

$$f'(-3) = \lim_{h \to 0} \frac{f(-3+h) - f(-3)}{h}$$
$$= \lim_{h \to 0} \frac{(-3+h)^2 - (-3)^2}{h}$$
$$= \lim_{h \to 0} \frac{9 - 6h + h^2 - 9}{h}$$
$$= \lim_{h \to 0} \frac{h(h-6)}{h}$$
$$= -\frac{6}{h}$$

Example

Let's find f'(-3) using the second definition of the derivative.

Solution:

$$f'(-3) = \lim_{x \to -3} \frac{f(x) - f(-3)}{x+3}$$
$$= \lim_{x \to -3} \frac{x^2 - (-3)^2}{x+3}$$
$$= \lim_{x \to -3} \frac{x^2 - 9}{x+3}$$
$$= \lim_{x \to -3} \frac{(x-3)(x+3)}{(x+3)}$$
$$= -6$$

Example

Let's find the derivative of $f(x) = x^2$ at an arbitrary value for x.

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Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h \to 0} \frac{(x+h-x)(x+h+x)}{h}$$
$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$
$$= 2x$$

Example

Let's consider the following example. Find f'(t) for the function $f(t) = \sqrt{t}, t \ge 0$.

Solution:

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{t+h} - \sqrt{t}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{t+h} - \sqrt{t}}{h}\right) \left(\frac{\sqrt{t+h} + \sqrt{t}}{\sqrt{t+h} + \sqrt{t}}\right)$$

$$= \lim_{h \to 0} \frac{t+h-t}{h(\sqrt{t+h} + \sqrt{t})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{t+h} - \sqrt{t})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{t+h} + \sqrt{t}}$$

$$= \frac{1}{2\sqrt{t}}, t > 0$$

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Worksheet $\#1$	Derivative	Differential Calculus

Exercises

Use the definition of the derivative to determine the derivative.

a)
$$f(x) = x^2 + 3x$$
 e) $y = c$

b) $f(x) = \frac{3}{x+2}$

g) y = mx + b, where m and b are constants

f) y = x

d) $f(x) = \frac{1}{x^2}$

c) $f(x) = \sqrt{3x+2}$

h) $y = ax^2 + bx + c$, where a, band c are constants.

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