Graph Sketching

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2021

Graph Sketching

Using everything you have learned about the derivative of a function, we're going to put it all together to sketch the graph of the curve represented by a function.

Algorithm for graph sketching

1. Find discontinuities of $f(x)$. e.g. when $f(x) = \pm\infty$.
2. Find the asymptotes, vertical, horizontal and oblique.
3. Determin the intercepts, y and x intercepts.
4. Determine the critical pionts from $f'(x) = 0$.
5. Determinen concavity using $f''(x)$.
6. Determine inflection points from $f''(x) = 0$.
7. Sketch graph using above information.

Example

Sketch $y = x^3 - 3x^2 - 9x + 10$

Solution:

$$\begin{aligned} y' &= 3x^2 - 6x - 9 \\ y'' &= 6x - 6 \\ y = 0 &= 3x^2 - 6x - 9 \\ 0 &= x^2 - 2x - 3 \\ 0 &= (x-3)(x+1) \end{aligned}$$

Therefore, $c = 3 - 1$.

$$\begin{aligned} y''(3) &= 18 - 6 = 12 > 0 \\ y''(-1) &= -6 - 6 = -12 < 0 \end{aligned}$$

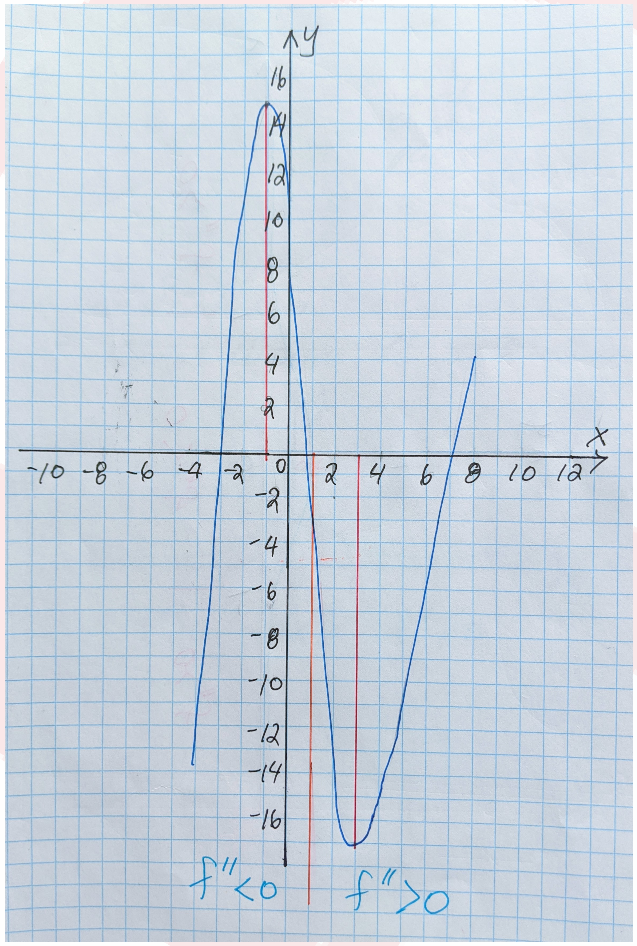
Therefore the function is concave up at $c = 3$ and concave down at $c = -1$. Where does the function change concavity, or where is the inflection point?

$$y'' = 6x - 6 = 0 \Rightarrow x = 1$$

Therefore when $x = 1 = c$ we have an inflection point. Let's now find the points at which the function has a minimum, maximum and inflection point.

$$\begin{aligned} y(3) &= 27 - 27 - 27 + 10 = -17 \\ y(-1) &= -1 - 3 + 9 + 10 = 15 \\ y(1) &= 1 - 3 - 9 + 10 = -1 \end{aligned}$$

Therefore, $(3, -17), (-1, 15)$ and $(1, -1)$ are the minimum, maximum and inflection points, respectively, of the function f.



y
x
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-10 -8 -6 -4 -2
2 4 6 8 10 12
f''<0
f''>0

Exercises

Sketch each of the following,

a) $y = x^3 - 9x^2 + 15x + 30$

b) $f(x) = x^4 - 4x^3 - 8x^2 + 48x$

c) $y = 3 + \frac{1}{(x+2)^2}$

d) $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

e) $y = \frac{6x^2-2}{x^3}$

f) $y = \frac{x^2-3x+6}{x-1}$