Introduction to Factoring Polynomials (Single Variable)

RaiseMyMarks.com

2021

Factoring polynomials

When you learnt how to multiply, you got so good at it that you then were asked to find the "factors" of a particular number. For example, the number 36. What are all the factors of 36? When I see this kind of problem I think, "What numbers multiply together to give 36?" Let's start with 1.

$$\begin{aligned} 1 \times 36 &= 36 \\ 2 \times 18 &= 36 \\ 3 \times 12 &= 36 \\ 4 \times 9 &= 36 \\ 6 \times 6 &= 36 \end{aligned}$$

We can go even further and write 36 as the prodcut of more than 2 numbers.

$$\begin{aligned} 1 \times 3 \times 2 \times 3 \times 2 &= 36 \\ 2 \times 2 \times 9 &= 36 \\ 3 \times 3 \times 4 &= 36 \\ 2 \times 2 \times 3 \times 3 &= 36 \\ 2 \times 3 \times 2 \times 3 &= 36 \\ 2 \times 2 \times 3 \times 3 &= 36 \\ 3 \times 3 \times 2 \times 2 &= 36 \end{aligned}$$

So the factors of 36 are,

$$1 \text{ and } 36,\ 2 \text{ and } 18,\ 3 \text{ and } 12,\ 4 \text{ and } 9,\ 6 \text{ and } 6$$

So we have rewrtiten 36 as a product of its factors. We can do the same for polynomials. The goal when factoring polynomials is to try and rewrite the polynomials as a product of "smaller" polynomials. What

does a"smaller" polynomial mean? In this case it means a polynomial of lesser or smaller"degree" than the oiginal polynomial we were given.

We'll stick to one variable in this introduction. Now, let's consider some examples.

Example

Find all factorizations for the following polynomials:

1. x^2
2. x^4
3. $x^2 + 2x$
4. $x^3 + 2x - 3x^2$
5. $x^2 + x^6 + x^8 + x^{12}$

Solution:

1.

$$\begin{aligned} x^2 &= x^2 \cdot 1 \\ &= x \cdot x \end{aligned}$$

2.

$$\begin{aligned} x^4 &= x^4 \cdot 1 \\ &= x^3 \cdot x \\ &= x^2 \cdot x^2 \end{aligned}$$

3.

$$x^2 + 2x = x(x + 2)$$

4.

$$x^3 + 2x - 3x^2 \;=\; x(x^2 + 2 - 3x)$$

5.

$$x^2 + x^6 + x^8 + x^{12} \;=\; x^2(1 + x^4 + x^6 + x^{10})$$

Exercises

Factor each polyonmial.

1. $x^2 + x^3$
2. $3x^2 + 6x^4 - 4x^4$
3. $x(3 + x^2) - x^3(3 + x^2)$
4. $-x + 2x$
5. $-3x^2 - 5x + 8x^7$
6. $5x^8 - 5x^7 + 8x^7$
7. $x^3 + x^2 + x$
8. $2x^{10} - 2x^8$
9. $x^2 + x^{10} + x^{18} + x^{26}$
10. $-6x^5 + 3x^4 - x^3 + 2x^2$