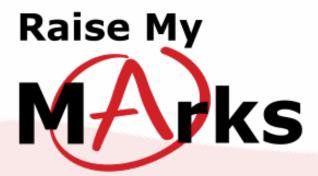
Properties of Logarithms



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2021

Properties of logarithm

Let's consider some properties of the logarithmic function,

$$y = f(x) = \log_b x \tag{1}$$

- 1. b > 0
- 2. x-intercept = 1
- 3. y-axis is a vertical asymptote
- 4. Domain= $\{x \in \mathbb{R} | x > 0\}$
- 5. Range = $\{y|y \in \mathbb{R}\}$
- 6. If b > 1 then the logarithmic function is increasing.
- 7. If 0 < b < 1 then the logarithm function is decreasing.

Notes, the most common base used is 10 for the logarithm function. This logarithm is writtedn as $\log x$ rather than $\log_{10} x$. The value of the base b can be omitted when b = 10.

Some basic properties of logarithms

- 1. $\log_b 1 = 0$
- 2. $\log_b b = 1$
- $3. \log_b b^x = x$
- 4. $b^{\log_b x} = x$

More properties of logarithms when x > 0, w > 0 and $r \in \mathbb{R}$ is a real number.

5.

$$\log_a(xw) = \log_a x + \log_a w$$

6.

$$\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$$

7.

$$\log_a x^r = r \log_a x$$

Let's use some of these properties to solve logarithmic equation.

Example

Solve $\log_6 x = 2$.

Solution: $\log_6 x = 2 \text{ means } x = 6^2 = 36.$

Example

Solve
$$\log_6 x + \log_6(x+1) = 1$$

Solution:

$$\log_6 x + \log(x+1) = 1$$
, multiplicative property $\log_6[(x(x+1))] = 1$ equivalence to exponential $6^1 = x(x+1)$ $0 = x^2 + x - 6$ $0 = (x+3)(x-2)$

Therefore, x = -3 or 2.

Example

Solve $3^x = 23$.

Solution:

$$3^x = 23$$
, Take log base 10 on both sides $\log 3^x = \log 23$, power property $x \log 3 = \log 23$, solve for x $x = \frac{\log 23}{\log 3}$

Exercises

1. Solve.

(a)
$$\log_5 x = 3$$

(b)
$$\log_4 x = 2$$

(c)
$$\log_4\left(\frac{1}{64}\right) = x$$

(d)
$$\log_4 x = 2$$

(e)
$$\log_{\frac{1}{4}} x = -2$$

$$(f) \log_x 27 = 3$$

2. Sketch on one graph each of the following:

(a)
$$y = 5^x$$
 and $y = \log_5 x$

(b) $y = 5^{-x} \text{ and } y = \log_{\frac{1}{5}} x$