Properties of Logarithms

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Properties of logarithm

Let's consider some properties of the logarithmic function,

$$y = f(x) = \log_b x \tag{1}$$

1. $b > 0$
2. $x-$intercept $= 1$
3. $y-$axis is a vertical asymptote
4. Domain$= \{x \in \mathbb{R} | x > 0\}$
5. Range $= \{y | y \in \mathbb{R}\}$
6. If $b > 1$ then the logarithmic function is increasing.
7. If $0 < b < 1$ then the logarithm function is decreasing.

Notes, the most common base used is 10 for the logarithm function. This logarithm is writtedn as $\log x$ rather than $\log_{10} x$. The value of the base b can be omitted when $b = 10$.

Some basic properties of logarithms

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

More properties of logarithms when $x > 0, w > 0$ and $r \in \mathbb{R}$ is a real number.

5.
$$\log_a(xw) = \log_a x + \log_a w$$

6.
$$\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$$

7.
$$\log_a x^r = r \log_a x$$

Let's use some of these properties to solve logarithmic equation.

Example

Solve $\log_6 x = 2$.

Solution: $\log_6 x = 2$ means $x = 6^2 = 36$.

Example

Solve $\log_6 x + \log_6(x+1) = 1$

Solution:

$$\begin{aligned}
\log_6 x + \log(x+1) &= 1, \quad \text{multiplicative property} \\
\log_6[(x(x+1))] &= 1 \;\; \text{equivalence to exponential} \\
6^1 &= x(x+1) \\
0 &= x^2 + x - 6 \\
0 &= (x+3)(x-2)
\end{aligned}$$

Therefore, $x = -3$ or 2.

Example

Solve $3^x = 23$.

Solution:

$$\begin{aligned} 3^x &= 23, \quad \text{Take log base 10 on both sides} \\ \log 3^x &= \log 23, \quad \text{power property} \\ x \log 3 &= \log 23, \quad \text{solve for x} \\ x &= \frac{\log 23}{\log 3} \end{aligned}$$

Exercises

1. Solve.

   (a) $\log_5 x = 3$

   (b) $\log_4 x = 2$

   (c) $\log_4 \left(\frac{1}{64}\right) = x$

   (d) $\log_4 x = 2$

   (e) $\log_{\frac{1}{4}} x = -2$

   (f) $\log_x 27 = 3$

2. Sketch on one graph each of the following:

   (a) $y = 5^x$ and $y = \log_5 x$

   (b) $y = 5^{-x}$ and $y = \log_{\frac{1}{5}} x$