

Properties of Logarithms

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Properties of logarithm

Let's consider some properties of the logarithmic function,

$$y = f(x) = \log_b x \quad (1)$$

1. $b > 0$
2. x -intercept = 1
3. y -axis is a vertical asymptote
4. Domain = $\{x \in \mathbb{R} | x > 0\}$
5. Range = $\{y | y \in \mathbb{R}\}$
6. If $b > 1$ then the logarithmic function is increasing.
7. If $0 < b < 1$ then the logarithm function is decreasing.

Notes, the most common base used is 10 for the logarithm function. This logarithm is written as $\log x$ rather than $\log_{10} x$. The value of the base b can be omitted when $b = 10$.

Some basic properties of logarithms

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

More properties of logarithms when $x > 0$, $w > 0$ and $r \in \mathbb{R}$ is a real number.

5.

$$\log_a(xw) = \log_a x + \log_a w$$

6.

$$\log_a\left(\frac{x}{w}\right) = \log_a x - \log_a w$$

7.

$$\log_a x^r = r \log_a x$$

Let's use some of these properties to solve logarithmic equation.

Example

Solve $\log_6 x = 2$.

Solution: $\log_6 x = 2$ means $x = 6^2 = 36$.

Example

Solve $\log_6 x + \log_6(x + 1) = 1$

Solution:

$$\begin{aligned}\log_6 x + \log(x + 1) &= 1, && \text{multiplicative property} \\ \log_6[(x(x + 1))] &= 1 && \text{equivalence to exponential} \\ 6^1 &= x(x + 1) \\ 0 &= x^2 + x - 6 \\ 0 &= (x + 3)(x - 2)\end{aligned}$$

Therefore, $x = -3$ or 2 .

Example

Solve $3^x = 23$.

Solution:

$$3^x = 23, \text{ Take log base 10 on both sides}$$

$$\log 3^x = \log 23, \text{ power property}$$

$$x \log 3 = \log 23, \text{ solve for x}$$

$$x = \frac{\log 23}{\log 3}$$

Exercises

Evaluate.

(a) $\log_2 8$

(b) $\log_3 \left(\frac{1}{27}\right)$

(c) $\log_2 4^2$

(d) $\log_5 \sqrt{5}$

(e) $\log_3(9 \times \sqrt[5]{9})$

(f) $\log_2 16^{1/3}$

$$(g) \log_6 \sqrt{36} - \log_{25} 5$$

$$(h) \log_7 49 + \log_2 64$$

$$(i) \log_9 \left(\frac{1}{3}\right) + \log_3 \left(\frac{1}{9}\right)$$

$$(j) \log_2 \sqrt[4]{32}$$