

Logarithms - Change of Base

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2021

Change of Base

Sometimes we will be faced with logarithmic or exponential equations where the bases are not the same. Being able to change from one base to another will be helpful in these situations. Let's have a look at the change of base formula for logarithmic functions.

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b} \quad (1)$$

Note:

$$\log_t b = \frac{1}{\log_b t}$$

Let's try and prove the change of base formula.

Prove: Prove the following formula,

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Proof:

$\log_b x = y$, this is equivalent to

$b^y = x$, let's consider using a different base a and see what we get

$$\log_a(b^y) = \log_a x$$

$$y \log_a b = \log_a x$$

$$y = \frac{\log_a x}{\log_a b}$$

$$\therefore \log_b x = \frac{\log_a x}{\log_a b}$$

Now that we have proved the change of base formula let's try some examples.

Example

Show that,

$$\frac{1}{\log_3 a} + \frac{1}{\log_4 a} = \frac{1}{\log_{12} a}$$

Solution:

$$\frac{1}{\log_3 a} + \frac{1}{\log_4 a} = \frac{1}{\log_{12} a}$$

$$L.S. = \log_a 3 + \log_a 4, \quad \text{by the Note from the change of base formula above}$$

$$= \log_a(3 \times 4)$$

$$= \log_a 12$$

$$R.S. = \frac{1}{\log_1 2a} = \log_a 12 = L.S.$$

Example

If $a^2 + b^2 = 14ab$, $a > 0$, $b > 0$, show that,

$$\log \left(\frac{a+b}{4} \right) = \frac{1}{2}(\log a + \log b)$$

Solution:

$$a^2 + b^2 = 14ab$$

$$a^2 + 2ab + b^2 = 14ab + 2ab$$

$$(a + b)(a + b) = 16ab$$

$$(a + b)^2 = 16ab$$

$$\frac{(a + b)^2}{16} = ab$$

$$\left(\frac{a + b}{4}\right)^2 = ab$$

$$\log\left(\frac{a + b}{4}\right)^2 = \log(ab)$$

$$2\log\left(\frac{a + b}{4}\right) = \log a + \log b$$

$$\therefore \log\left(\frac{a + b}{4}\right) = \frac{1}{2}(\log a + \log b)$$

Exercises

Prove the following,

(a)

$$\frac{1}{\log_5 a} = \frac{1}{\log_2 a} = \frac{1}{\log_{15} a}$$

(b)

$$\frac{2}{\log_8 a} - \frac{4}{\log_2 a} = \frac{1}{\log_4 a}$$

(c)

$$\frac{2}{\log_6 a} = \frac{1}{\log_{36} a}$$

(d) If $a^2 + b^2 = 23ab$, $a > 0, b > 0$. Show, $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$.

- (e) For $a > 0, a \neq 1, x > 0$ prove that $\log_a \frac{1}{x} = \log_{\frac{1}{a}} x$.
- (f) If $\log_a b = p^3$ and $\log_b a = \frac{4}{p^2}$. Show that $p = \frac{1}{4}$.
- (g) If $a^3 - b^3 = 3a^2b + 5ab^2, a > 0, b > 0$ show, $\log\left(\frac{a-b}{2}\right) = \frac{1}{3}(\log a + 2\log b)$
- (h) For $a > 1, b > 1$, show $(\log_a b)(\log_b a) = 1$.