Applications of Logaritmic Functions

RaiseMyMarks.com

2021

Where can logarithms be used?

Logarithms and EARTHQUAKES

Logarithms can be used when measuring the magnitude of an earthquake. The Richter formula is given by,

$$M = \log\left(\frac{I}{I_0}\right)$$

where M = Richter number, I = intensity of earthquake being measured. I_0 = intensity of a reference earthquake.

Example

An earthquake of magnitude 7.5 on the Richter scale struck Guatemala on February 4, 1976 killing 23000 people. On Oct 2, 1993, an earthquake of magnitude 6.4 killed 20000 in Maharashtra, India. Compare the intensitites of the two earthquakes.

Solution I_G = Guatemala earthquake intensity; I_i = Indian earthquake intensity; $M_G = 7.5$, $M_i = 6.4$. We need to determine the intensities of the two earthquakes relative to the reference I_0 and then

to each other.

$$\begin{aligned} M &= \log\left(\frac{I}{I_0}\right) \\ M_G &= \log\left(\frac{I_g}{I_0}\right) \\ 7.5 = \log\left(\frac{I_g}{I_0}\right) &\Leftrightarrow 10^{7.5} = \left(\frac{I_G}{I_0}\right) \\ & \quad 10^{7.5} I_0 = I_G \\ M_i &= \log\left(\frac{I_i}{I_0}\right) \\ 6.4 = \log\left(\frac{I_i}{I_0}\right) &\Leftrightarrow 10^{6.4} = \left(\frac{I_i}{I_0}\right) \\ & \quad 10^{6.4} I_0 = I_i \\ \therefore \frac{I_g}{I_i} &= \frac{10^{7.5} I_0}{10^{6.4} I_0} \\ \text{the intensity of the Guatemala earthquake is } I_G &= 10^{7.5-6.4} I_i \\ \text{and the intensity of the Indian earthquake is } I_G &= 10^{1.1} I_i \end{aligned}$$

Logarithms and SOUND

Logarithms cadn also be used in sound. The formula to compare sound is,

$$L = 10 \log\left(\frac{I}{I_0}\right),$$

where I =intensity of sound being measured; I_0 = intensity of sound at threshold of hearing, L+loudness measured in decibels (1/10 of a bel).

Note: At the threshold of hearing, the loudness of sound is 0 dB.

Let's consider and example.

Example

A sound is 1000 times more intense than a sound you can just hear. What is the measure of its loudness in decibels?

Solution

$$\begin{aligned} I &= 1000I_0 \\ L &= 10\log\left(\frac{I}{I_0}\right) \\ &= 10\log\left(\frac{1000I_0}{I_0}\right) \\ &= 10\log(1000) \\ &= 10\log 10^3 \\ &= (10)(3)\log 10 \\ &= (10)(3)(1) \\ L &= 30 \end{aligned}$$

Therefore, the loudness of the sound is 30 dB.

Logarithms and CHEMISTRY

Logarithms can be used in chemistry. The acidity of a liquid on a pH scale is given by,

$$pH = -\log[H^+]$$

where, $[H^+]$ = concentration of the hydrogen ion in moles per litre. For distilled water, $[H^+] = 10^{-7} mol/L$. The pH of distilled water is,

$$\begin{aligned} pH &= -\log[H^+] \\ &= -\log(10^{-7}) \\ &= -(-7)\log 10 \\ &= 7 \end{aligned}$$

Distilled water is the *benchmark*, so a pH of 7 is the value to compare against.

$$\begin{aligned} pH < 7 &\Leftrightarrow \text{liquid is an acid} \\ pH > 7 &\Leftrightarrow \text{liquie is a base} \end{aligned}$$

Let's consider an example.

Example

The pH of a fruist juice is 3.10. What is the hydrogen ion concentration of the fruit juice?

Solution

$$\begin{aligned} pH &= -\log[H^+] \\ 3.10 &= -\log[H^+] \\ -3.10 &= log[H^+] \\ 10^{-3.10} &= [H^+] \end{aligned}$$

Exercises

1. If one earthquake has a magnitude of 5 on the Richter scale and a second earthquake has a magnitude of 6, compare the intensities of the two earthquakes.

2. A sound is 1 000 000 times more intense than a sound you can just hear. What is the loudess of the sound?

3. Find the pH of a liquid with a hydrogen ion concentration of $8.7 \times 10^{-6} mol/L$?

4. An earthquake of magnitude 2 cannot be felt. An earthquake of magnitude 4 will be noticed but usually causes no damage. Compare the intensities of two such earthquakes.

5. An earthquake in Gansu, China, on Decmeber 16 1920 measured 8.6 on the Richter scale and killed 100 000 people. An earthquake that usually causes no damage measure 4 on the Richter scale. Compare the intensities of the two earthquakes.

6. On January 24, 1939, an earthquake measureing 8.3 occured in Chillan, Chile, killing 28 000 people On September 21, 1999, an earthquake in Taiwan measure 7.6 on the Richter scale and killed 2100 people. Copare the intensities of these two earthquakes.

7. Sash need a new muffler on her car. She has been told that the sound from her car was measured at 120 dB. After installing the new muffler, the loudness of her car is 75 dB. How many times more intense was the sound from her defective muffler?

8. Tania's infant daughter has colic and cries during the night. The noise level in the house at these times is 75 dB. When the baby finally falls asleep, the noise level is 35 dB. How many times mkore intense if the noise level in the house when the baby is crying?

9. Jonathan lives near a busy street. he has all the windows in his home open and measures the hoise level inside as 79 dB. He closes the windows and finds the noise level is 68 dB. By what factor did the intensity of the noise ecrease when Jonathan closed the windows?

10. Find the hydrogen ion concentration of milk, which has a pH of 6.50.

11. Find the hydrogen ion concentration of milk okf magnesia, which has a pH of 10.50.

