Dividing Polynomials

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Division of Polynomials

Division of polynomials is similar to division of numbers. For example, $536 \div 8$,

$$\begin{array}{r} 67 \\ 8\overline{)536} \\ \underline{48} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

So, $8 \times 67 = 536$.

Example

Let's try another example, $247 \div 4$,

$$\begin{array}{r} 61 \\ 4\overline{)247} \\ \underline{24} \\ 07 \\ \underline{04} \\ 3 \end{array}$$

So, $4 \times 61 + 3 = 244 + 3$ where $d = 4$ is the divisor, $1 = 61$ the quotient and $r = 3$ the remainder. Division of polynomials is the same,

$$f(x) = d(x)q(x) + r(x)$$

where,

$$\begin{aligned} d(x) &= \text{divisor polynomial} \\ q(x) &= \text{quotient polynomial} \\ r(x) &= \text{remainder polynomial} \end{aligned}$$

Example

Let's consider an example. $f(x) = x^2 - 7x = 10$ and $d(x) = x + 2$. So we're dividing $f(x)$ by $d(x)$.

$$
\begin{array}{r|l}
 & x - \ \ 9 \\ \hline
x+2 \,) & x^2 - \ 7x \ - 10 \\
 & -(x^2 + \ 2x) \\ \cline{2-2}
 & \qquad -9x - 10 \\
 & \qquad -(-9x - 18) \\ \cline{2-2}
 & \qquad\qquad 8
\end{array}
$$

Now we have, $q(x) = x - 9$ and $r(x) = 8$ or

$$x^2 - 7x = 10 = (x + 2)(x - 9) + 8$$

Example

Let's consider another example,

$$(3x^4 - 2x^3 + 4x^2 - 7x + 4) \div (x^2 - 3x + 1)$$

Solution:

$$
\begin{array}{r|l}
 & 3x^2 + \ 7x \ + \ \ 22 \\ \hline
x^2 - 3x + 1 \,) & 3x^4 - 2x^3 + 14x^2 - \ 7x + \ 4 \\
 & -(3x^4 - 9x^3) + \ 3x^2 \\ \cline{2-2}
 & \qquad 7x^3 + \ \ x^2 \ - \ 7x \\
 & \qquad -(7x^3 - 21x^3 + 7x) \\ \cline{2-2}
 & \qquad\qquad 22x^2 - 14x + \ 4 \\
 & \qquad\qquad -(22x^2 - \ 66 + 22) \\ \cline{2-2}
 & \qquad\qquad\qquad 52x - 18
\end{array}
$$

No we have, $q(x) = 3x^2 + 7x + 22$ and $r(x) = 52x - 18$ or

$$3x^4 - 2x^3 + 4x^2 - 7x + 4 = (x^2 - 3x + 1)(3x^2 + 7x + 22) + (52x - 18)$$

Exercises

1. When a polynomial is divided by $x-3$ its quotient is x^2-5x-7 and its remainder is 5. What is the polynomial?

2. When a polynomial is divided by x^2+x+1 its quotient is x^2-x+1 and is remainder is -1. What is the polynomial?

3. Divide then write the answer in the form $f(x)=d(x)q(x)+r(x)$.
   (a) $(x^3-3x^2+x+2)\div(x+2)$

   (b) $(3x^3+x^2+2x-1)\div(3x-1)$

   (c) $(-6x^4+3x^3-x^2+2x+2)\div(x-4)$

(d) $(3x^3 + 7x + 5x + 1) \div (3x + 1)$

(e) $(x^4 - 2x^3 + 3x^2 - 3x + 8) \div (x - 2)$

(f) $(x^5 - 1) \div (x - 1)$

(g) $(4x^3 + 32) \div (x + 2)$

(h) $(x^3 + 3x^2 - 16x + 12) \div (x - 2)$

(i) $(x^4 - 5x^2 + 4) \div (x^2 - 3x + 2)$

4. The condition for $d(x)$ to be a factor of $f(x)$ is $r(x) = 0$. For the exercises in # 3 which divisors $d(x)$ are factors of $f(x)$?

