

Dividing Polynomials

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Polynomial Long Division

When factoring polynomials we are in some sense dividing one polynomial into another. How do you divide polynomials. Good question and let's consider an example to go through the steps.

Example

Let's try dividing $x^4 + x^3 - 11x^2 - 5x + 6$ by $x + 3$.

Solution

$$\begin{array}{r}
 x^3 - 2x^2 - 5x + 2 \\
 \hline
 x + 3 \) \ x^4 + x^3 - 11x^2 - 5x + 6 \\
 \underline{-(x^4 + 3x^3)} \\
 -2x^3 - 11x^2 \\
 \underline{-(-2x^3 - 6x^2)} \\
 -5x^2 - 5x \\
 \underline{-(-5x^2 - 15x)} \\
 10x + 6 \\
 \underline{-(10x + 6)} \\
 0
 \end{array}$$

After dividing $x+3$ into $x^4+x^3-11x^2-5x+6$ we get x^3-2x^2-5x+2 which tells us that x^3-2x^2-5x+2 and $x+3$ are both factors of $x^4+x^3-11x^2-5x+6$. Or,

$$x^4 + x^3 - 11x^2 - 5x + 6 = (x + 3)(x^3 - 2x^2 - 5x + 2).$$

Let's consider another example that doesn't work out quite as well.

Example

Divide $10x^4 - 50x^3 - 800$ by $x - 6$ using long division.

Solution

$$\begin{array}{r}
 10x^3 + 10x^2 + 60x + 360 \\
 \hline
 x - 6 \) \ 10x^4 - 50x^3 - 800 \\
 \underline{-(10x^4 - 60x^3)} \\
 10x^3 - 800 \\
 \underline{-(10x^3 - 60x^2)} \\
 60x^2 - 800 \\
 \underline{-(60x^2 - 360x)} \\
 360x - 800 \\
 \underline{-(360x - 2160)} \\
 1360
 \end{array}$$

Therefore,

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 - \frac{1360}{x - 6}$$

Exercises

Divide the following,

(a) $2x^2 - 4x + 1 \div x - 2$

(b) $\frac{x(x+1)-4(x+2)}{x+1}$

(c) $\frac{2x^3+5x^2-4x+2}{2x+1}$

(d) $\frac{x^6+3x^4-x^2+2}{x^2+2x}$

(e) $\frac{3x^3-5x^2+10x-3}{3x+1}$

(f) $2x^3 - 9x^2 + 15 \div 2x - 5$

(g) $\frac{4x^4+1+3x^3+2}{x^2+x+2}$

(h) $\frac{2x^2-2+2x}{x+4}$

(i) $\frac{x^2-3x-10}{x+2}$

(j) $x^6 + 2x^4 + 6x - 9 \div x^3 + 3$