Piecewise Functions

RaiseMyMarks.com

2021

Piecewise Functions

We know what a function is but what is apiecewise function? A piecewise function is a funciton by using two or more rules or pieces of functions on two or more intervals. As a result, the graph of the function is made up of two ore more pieces of similar or different functions. Let's consider an example.

Example The *absolute value function* is a piecewise function.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

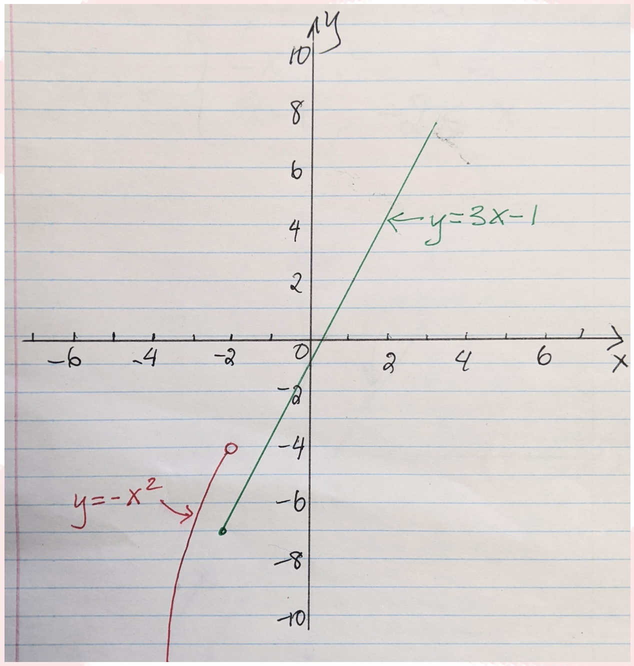
Notice that the real numbers $\mathbb{R}$ are divided up into two intervals or two pieces, $x < 0$ or $x \geq 0$. On each piece we have a different function. On the first "piece", $x < 0$, $f(x) = -x$; on the second piece, $0 \leq x$, the function is $f(x) = x$.

Example Graph the following piecewise function,

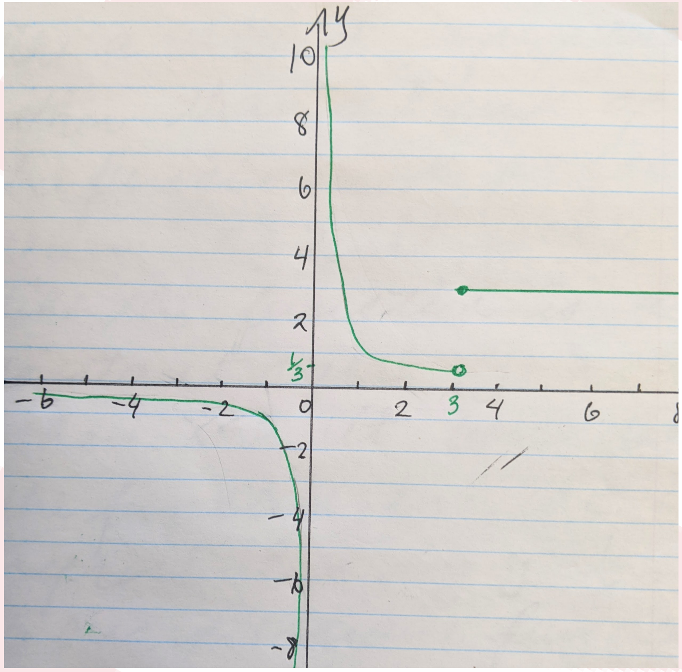
$$f(x) = \begin{cases} -x^2 & \text{if } x < -2 \\ 3x - 1 & \text{if } -2 \leq x \end{cases}$$

Solution

©Raise My Marks 2021



Example Given the following graph of a piecewise function, represent the function algebraically.



Solution

1. Let's divide the x-axis into pieces. It looks like there is a break in the function when x=0 and x=3. However, when x=0, looks like a vertical asymptote rather than a "break" of the function into "pieces".

2. When $x \geq 3$, it looks like the function is a constant value, $y = 3$. When $x < 3$, it looks like the function can be represented by $y = \frac{1}{x}$.

3. So, our piecewise function can be represented algebraically by,

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 3 \\ 3 & \text{if } 3 \leq x \end{cases}$$

Note: An open circle on a graph means the function approaches this point but never actually raches it. A closed circle means the function actually attains this value.

Exercises

1. Graph the following piecewise functions,

   (a)

$$f(x) = \begin{cases} -2 & \text{if } x < 2 \\ 5 & \text{if } 2 \leq x \end{cases}$$

   (b)

$$f(x) = \begin{cases} -x+3 & \text{if } x < -1 \\ x-3 & \text{if } -1 \leq x \end{cases}$$

   (c)

$$f(x) = \begin{cases} x^2+2 & \text{if } x < 1 \\ 3x+1 & \text{if } 1 \leq x \end{cases}$$

   (d)

$$f(x) = \begin{cases} 2x-1 & \text{if } x < -1 \\ -x+1 & \text{if } -1 \leq x < 1 \\ -5x+4 & \text{if } 1 \leq x \end{cases}$$

   (e)

$$f(x) = \begin{cases} |x+2| & \text{if } x \leq -1 \\ -x^2+2 & \text{if } -1 < x \end{cases}$$

   (f)

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x < 5 \\ x-1 & \text{if } 5 \leq x \end{cases}$$

©Raise My Marks 2021

(g)

$$f(x) = \begin{cases} 4 & \text{if } x < 4 \\ 2x & \text{if } 4 \leq x \end{cases}$$

(h)

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 4 \\ 2 & \text{if } 4 \leq x \end{cases}$$

(i)

$$f(x) = \begin{cases} 3x & \text{if } x < -2 \\ x^2 & \text{if } -2 \leq x < 1 \\ -2x+1 & \text{if } 1 \leq x < 3 \\ \sqrt{x} & \text{if } 3 \leq x \end{cases}$$

2. Which of the functions in # 1 are continuous?

3. State the domain and range of each function in # 1.