Piecewise Functions

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Piecewise Functions

We know what a function is but what is apiecewise function? A piecewise function is a funciton by using two or more rules or pieces of functions on two or more intervals. As a result, the graph of the function is made up of two ore more pieces of similar or different functions. Let's consider an example.

Example The *absolute value function* is a piecewise function.

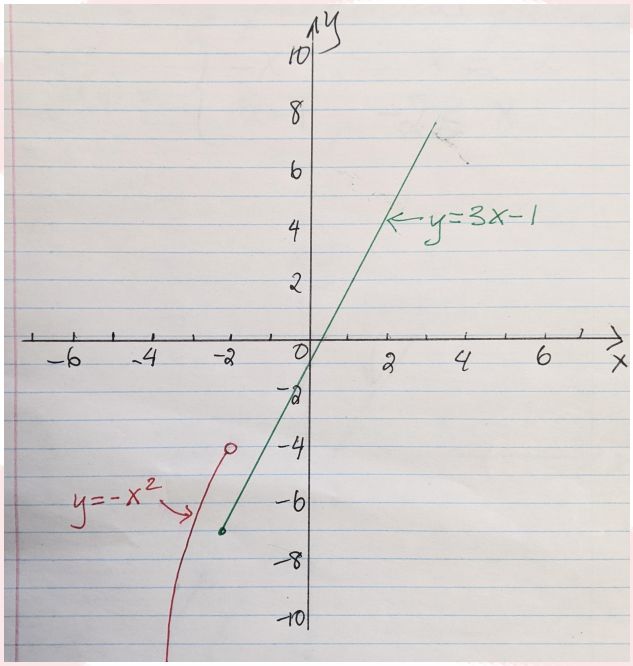
$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Notice that the real numbers $\mathbb{R}$ are divided up into two intervals or two pieces, $x < 0$ or $x \geq 0$. On each piece we have a different function. On the first "piece", $x < 0$, $f(x) = -x$; on the second piece, $0 \leq x$, the function is $f(x) = x$.

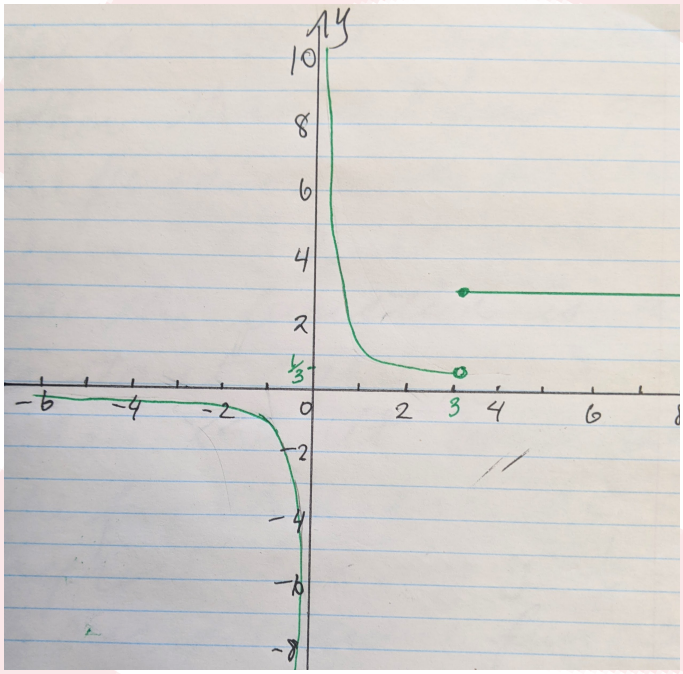
Example Graph the following piecewise function,

$$f(x) = \begin{cases} -x^2 & \text{if } x < -2 \\ 3x - 1 & \text{if } -2 \leq x \end{cases}$$

Solution



Example Given the following graph of a piecewise function, represent the function algebraically.



Solution

1. Let's divide the x-axis into pieces. It looks like there is a break in the function when x=0 and x=3. However, when x=0, looks like a vertical asymptote rather than a "break" of the function into "pieces".

2. When $x \geq 3$, it looks like the function is a constant value, $y = 3$. When $x < 3$, it looks like the function can be represented by $y = \frac{1}{x}$.

3. So, our piecewise function can be represented algebraically by,

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 3 \\ 3 & \text{if } 3 \leq x \end{cases}$$

Note: An open circle on a graph means the function approaches this point but never actually raches it. A closed circle means the function actually attains this value.

Exercises

1. For the function
$$f(x) = |x + 4|$$
   (a) Write $f(x)$ as a piecewise function.
   (b) Graph $f(x)$.

2. (a) Find the value of k that makes the following a continous function,
   (b) Graph the function.

$$f(x) = \begin{cases} -x^2 + k & \text{if } x \leq -1 \\ 2x + 1 & \text{if } -1 < x \end{cases}$$

3. The fish population, in tens of thousands, in a lake at any time g years is modelled by the following function,
$$f(t) = \begin{cases} 2^t & \text{if } 0 \leq t \leq 5 \\ 3t + 4 & \text{if } 5 < t \end{cases}$$
   This function describes the change in population due to an oil spill at time $t = 5$ years.
   (a) Sketch the graph of the function.
   (b) Describe the continuity of the function.
   (c) How many fish were killed by the oil spill?
   (d) When did the population recover to the level it was before the oil spill?

