Average Rate of Change

RaiseMyMarks.com

2020

Average Rate of Change

What does the average rate of change mean? Let's consider an example. Suppose have a table of values that represent the temperature every hour.

time hr	Temperature C	F
11	0	32
12	1	32
13	0	32
14	-1	30
15	-2	28
16	-3	27
17	-4	25
18	-4	25
19	-5	23
20	-5	23
21	-6	21
22	-6	21
23	-7	19
24	-8	18

The termperature is dependent on the time so temperature is a function of time or $T = T(t)$ is the dependent variable and t time is the independent variable. If we consider the change of the temperature over a period of time, say from 13 hours to 15 hours we have the following "rate of change"

$$\frac{T(15) - T(13)}{15 - 13} = \frac{28 - 32}{15 - 13} = \frac{-4}{2} = -2F/hr$$

then we can conclude the averae rate of change of the termperature is $-2F/hr$.

Average Rate of Change

For the funciton $y = f(x)$ the *average rate of change* of y with respect to x over the interval x_1 to x_2 is,

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Let's consider an example.

Example

The height of a model rocket in flight can be modelled by,

$$h(t) = -4.9t^2 + 32t + 1$$

where h is the hieght in metres and t seconds. What is the average rate of change of the rocket's height,

a) in the first second

b) between the 4^{th} and 5^{th} seconds.

Solution

a) The first second is the interval $0 \leq t \leq 1$. The rate of change of the height on this interval is given by,

$$\frac{\Delta h}{\Delta t} = \frac{h(1) - h(0)}{1 - 0} = \frac{-4.9 + 32 + 1 - 1}{1 - 0} = -4.9 + 32 = 27.1 m/s$$

b) The rate of change of the height on the interval $4 \leq t \leq 5$ is,

$$\begin{aligned} \frac{\Delta h}{\Delta t} &= \frac{h(5) - h(4)}{5 - 4} \\ &= \frac{-4.9(5^2 + 32(5) + 1 - ((-4.9(4^2) + 32(4) + 1)}{1} \\ &= \frac{38.5 - 52.2}{1} \\ &= -13.7 m/s \end{aligned}$$

Exercises

1. Determine the average rate of change of $g(x) = 4x^3 - 5x + 1$ on the intervals,
   (a) $3 \leq x \leq 4$
   (b) $4 \leq x \leq 5$
   (c) $4.5 \leq x \leq 5$
   (d) $4.75 \leq x \leq 5$

2. The volume of a cubic crystal grown in a laboratory is modelled by $V(x) = x^3$ where V is the volume in cubic centimetres and x is the side length. Find the average rate of change in the volume of the crystal w.r.t side length as each side grows from 4cm to 5cm.

3. A stone is dropped from a bridge that is 20ccm above a river. The table on the left gives the height of the falling stone above the water's surface. An algebraic model for the data is the polynomial function $h(t) = -4.9t^2+20$, where h is the height above the water, in metres, and t is the elapsed time in seconds and $t \geq 0$. Determine the average rate of change in height with respect to time over the first 2 seconds.

4. A skydiver jumps from an airplane. Before she opens her parachute, she is in free fall. The function $d(t) = 4.9t^2$ models the vertical distance, d, in metres she has travelled at t seconds. What does the rate of change of distance with respect to time represent? What units are used to measure this rate of change? What is the average rate of change of the vertical distance after 2 seconds?

5. Concentric cirlces from when a stone is dropped into a pool of water. What is the average rate of chage in the area of one circle with respect to radius as the radius grows from 0cm to 100cm?

