Average Rate of Change

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2020

Average Rate of Change

What does the average rate of change mean? Let's consider an example. Suppose have a table of values that represent the temperature every hour.

time hr	Temperature C	F
11	0	32
12	1	32
13	0	32
14	-1	30
15	-2	28
16	-3	27
17	-4	25
18	-4	25
19	-5	23
20	-5	23
21	-6	21
22	-6	21
23	-7	19
24	-8	18

The termperature is dependent on the time so temperature is a function of time or $T = T(t)$ is the dependent variable and t time is the independent variable. If we consider the change of the temperature over a period of time, say from 13 hours to 15 hours we have the following "rate of change"

$$\frac{T(15) - T(13)}{15 - 13} = \frac{28 - 32}{15 - 13} = \frac{-4}{2} = -2F/hr$$

then we can conclude the averae rate of change of the termperature is $-2F/hr$.

Average Rate of Change

For the funciton $y = f(x)$ the *average rate of change* of y with respect to x over the interval x_1 to x_2 is,

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Let's consider an example.

Example

The height of a model rocket in flight can be modelled by,

$$h(t) = -4.9t^2 + 32t + 1$$

where h is the hieght in metres and t seconds. What is the average rate of change of the rocket's height,

a) in the first second

b) between the 4^{th} and 5^{th} seconds.

Solution

a) The first second is the interval $0 \leq t \leq 1$. The rate of change of the height on this interval is given by,

$$\frac{\Delta h}{\Delta t} = \frac{h(1) - h(0)}{1 - 0} = \frac{-4.9 + 32 + 1 - 1}{1 - 0} = -4.9 + 32 = 27.1 m/s$$

b) The rate of change of the height on the interval $4 \leq t \leq 5$ is,

$$\begin{aligned} \frac{\Delta h}{\Delta t} &= \frac{h(5) - h(4)}{5 - 4} \\ &= \frac{-4.9(5^2 + 32(5) + 1 - ((-4.9(4^2) + 32(4) + 1)}{1} \\ &= \frac{38.5 - 52.2}{1} \\ &= -13.7 m/s \end{aligned}$$

Exercises

1. For $f(x) = x^2$, determine the average rate of change on the interval,
   (a) $1 \leq t \leq 6$
   (b) $1 \leq t \leq 2$
   (c) $1 \leq t \leq 1.5$
   (d) $-2 \leq t \leq 2$
   (e) $1 \leq t \leq 1.01$
   (f) $1 \leq t \leq 1.25$
   (g) $1 \leq t \leq 1.1$

2. For the function $f(x) = 2x^2 - 1$ complete the table below.

Interval	$\Delta f(x)$	Δx	$\frac{\Delta f(x)}{\Delta t}$
$1 \leq x \leq 2$			
$1.5 \leq x \leq 2$			
$1.75 \leq x \leq 2$			
$1.9 \leq x \leq 2$			
$1.95 \leq x \leq 2$			