Inverse Function

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Inverse Function

What is an inverse function? Let's consider an example to answer this question. Suppose you're given an equation,

$$d = 4.9t^2 + 3$$

where d represents the distance in metres an object falls from rest, $t = 0$, for t seconds. If we are given a time say$t = 3$, we can calculuate how far the object fell.

$$\begin{aligned} d &= 4.9(3)^2 + 3 \\ &= 4.6(9) + 3 \\ &= 47.1 \text{ metres} \end{aligned}$$

Suppose instead we are given the distance the object fell, say 81 metres, and we want to find out how long it took to fall that 81 metres. Now we have,

$$d = 81 = 4.9t^2 + 3$$

and we want to solve for t.

$$\begin{aligned} 81 &= 4.9t^2 + 3 \\ 81 - 2 &= 4.9t^2 \\ \frac{78}{4.9} &= t^2 \\ \pm\sqrt{\frac{78}{4.9}} &= t \end{aligned}$$

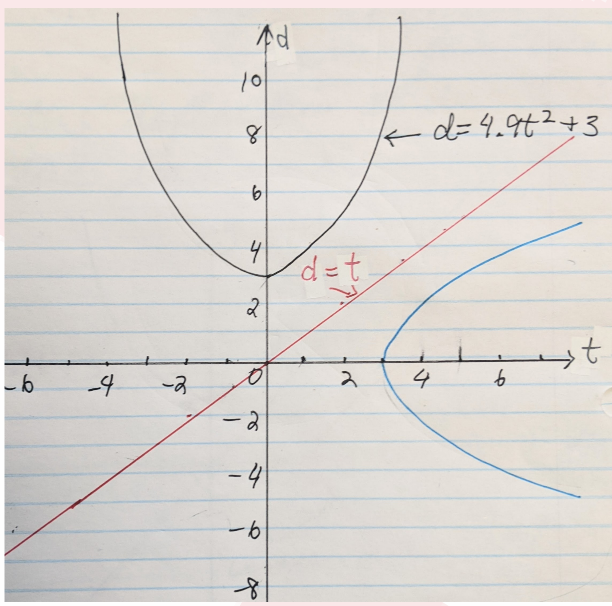
But tie can only be positive so we take the posotive square root given, $t = \sqrt{\frac{78}{4.9}}$ seconds.

The process to find t was the *inverse* of the process to find d. And if we were to repeat this process for any distance d, then we would get,

$$\begin{aligned} d &= 4.9t^2 + 3 \\ d - 3 &= 4.9t^2 \\ \frac{d-3}{4.9} &= t^2 \\ \pm\sqrt{\frac{d-3}{4.9}} &= t \end{aligned}$$

and t is called the *inverse function of d*.

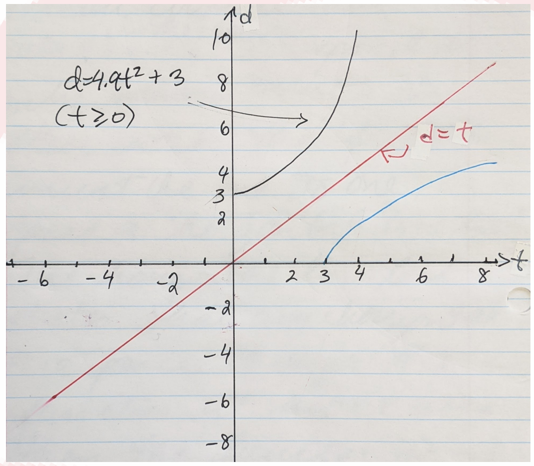
If the original function $d = 4.9t^2 + 3$ was graphed then the inverse function is the reflection of $d = 4.9t^2 + 3$ in the line $y = x$.



Notice that the inverse is not a function because it does not pass the vertical line test. In order to ensure the inverse is a function too we need to restrict the domain of the original function. For example, if we restrict the domain of $d = 4.9t^2 + 3$ to

$$\text{Domain} = \{x \in \mathbb{R} | x \geq 0\}$$

then the inverse is now a function and we have the graph,



In order for a function to have an inverse each x value in the domain of the function must correspond to only one y value. In other words, the function must be *one to one*, or pass a *horizontal line test*. The horizontal line test states that any horizontal line passes through the function at most once then the function is one to one.

How do you find the inverse? Good question. Let's consider an example.

Example Find the inverse of

$$f(x) = \frac{1}{x-2}$$

.

Solution

1. Interchange the positions of the x and y variables.

$$x = \frac{1}{y-2}$$

2. Solve for y.

$$\begin{aligned} y - 2 &= \frac{1}{x} \\ y &= \frac{1}{x} + 2 \end{aligned}$$

3. This new function

$$y = \frac{1}{x} + 2$$

is the inverse of the original function $f(x)$. This inverse function is denoted by,

$$f^{-1}(x) = \frac{1}{x} + 2$$

Exercises

Determine the inverse of each function algebraically. Restrict domains, whe necessary.

(a) $y = x + 4$

(b) $y = x^2$

(c) $y = \sqrt{x+3}$

(d) $y = 2x + 5$

(e) $y = 4x + 7$

(f) $y = \sqrt{x-1}$

(g) $y = \frac{2}{x}$

(h) $y = x^2 - 4$

(i) $y = (x-2)^2$

(j) $y = \frac{x^2}{2} - 1$