

Composition of Functions

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2021

What is the composition of functions?

The composition of functions means, one function is inserted into the another function where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x , or $x = g(t)$ for $f(x)$. Let's take a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the composition of two polynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \text{ and } g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution The composition of f and g looks like,

$$f \circ g(t) \text{ or } f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t + 1), \text{ where } x = g(t) = t + 1 \\ &= (t + 1)^2 + 3(t + 1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.

Let's try another example.

Example

Given $f(x) = 2 - x$ and $g(x) = \frac{2}{5-x}$ determine $f \circ g(x)$ and $g \circ f(x)$.

Solution: Let's start with $f \circ g(x)$.

$$\begin{aligned} f(g(x)) &= f\left(\frac{2}{5-x}\right) = 2 - \frac{2}{5-x} = \frac{2(5-x) - 2}{5-x} \\ &= \frac{10 - 2x - 2}{5-x} = \frac{8 - 2x}{5-x} \end{aligned}$$

Therefore, $f(g(x)) = \frac{8-2x}{5-x}$.

Now $g \circ f(x)$.

$$g \circ f(x) = g(2-x) = \frac{2}{5-(2-x)} = \frac{2}{5-2+x} = \frac{2}{3+x}$$

Therefore, $g(f(x)) = \frac{2}{3+x}$.

Exercises

1. Given $f(x) = x^2 - x$ and $g(x) = 4x + 1$,
 - (a) find $f(g(x))$ and
 - (b) find $g(f(x))$.

2. For each function determine f and g so that $y = f(g(x))$.
 - (a) $y = (x + 1)^2 - 2x(x + 1) + 3$

 - (b) $y = \frac{1}{2x+3}$

 - (c) $y = -2\sqrt{x - 1}$

 - (d) $y = (x^2 + 3)^4$

 - (e) $y = -(4x - 5)^3$

$$(f) y = \sqrt{x + 2}$$

$$(g) y = -\frac{1}{x+1}$$

$$(h) y = (x + 1)^2$$

$$(i) y = \sqrt{x^2 + 1}$$