# Sum and Difference of Functions

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# Sum and difference of functions

Let's start by looking at the sum and difference of two functions.

### Example

Let's consider an example. Let's consider the functions,

$$f(x) = x^2, \quad \text{and } g(x) = x$$

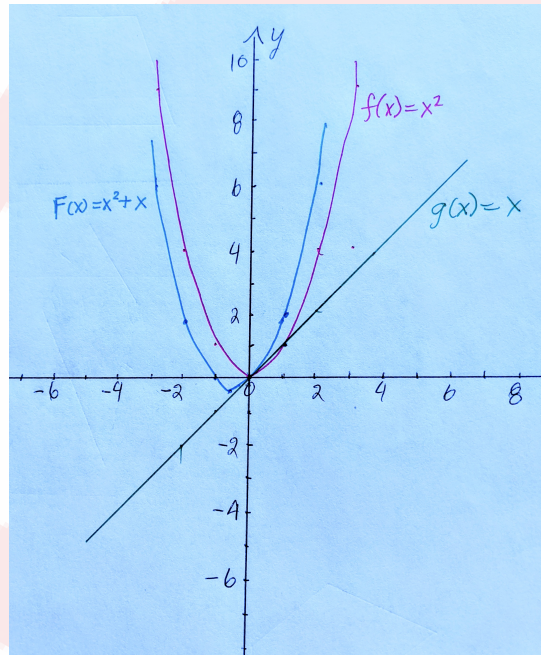Find the sum of $f$ and $g$ and then graph the sum.

### Sum of two functions

The sum of two functions is given by,

$$
\begin{aligned}
& f(x) + g(x) \\
= \ & x^2 + x \\
= \ & F(x)
\end{aligned}
$$

where $F(x)$ is a new function. Let's graph $F(x)$ now.

$$
\begin{aligned}
F(x) \ &= \ x^2 + x, \quad \text{is a quaratic} \\
&= \ x^2 + x + \frac{1}{4} - \frac{1}{4} \quad \text{completed square} \\
&= \ \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right) - \frac{1}{4} \\
&= \ \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}, \quad \text{vertex form of a quadratic}
\end{aligned}
$$

With the vertex form of the quadratic we can easily graph our function $F(x)$.

Let's consider another example.

## Example

Let's consider a difference of two functions. Let's consider the two functions,

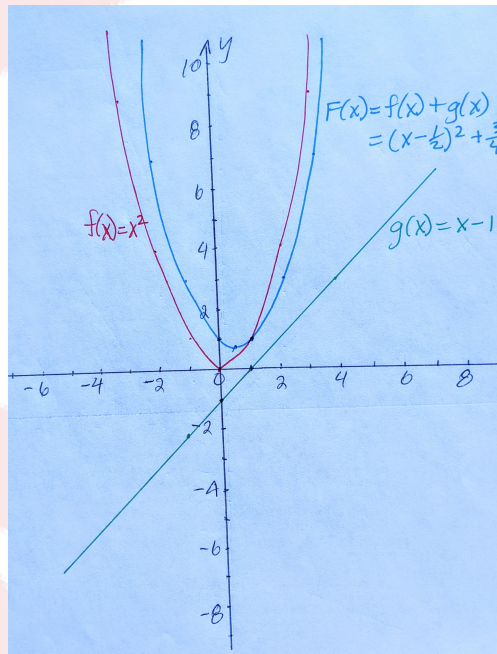$$f(x) = x^2 \quad \text{and} \quad g(x) = x + 1$$

Find the difference fof $f$ and $g$ and graph the resulting function.

## Difference of two functions

Our new function $F(x)$ is given by,

$$
\begin{aligned}
F(x) &= f(x) + g(x) \\
&= x^2 - (x + 1) \\
&= x^2 - x + 1, \quad \text{a quadratic} \\
&= x^2 - x + \frac{1}{4} - \frac{1}{4} + 1, \quad \text{completed square} \\
&= \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) + \frac{3}{4} \\
&= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}, \quad \text{vertex form}
\end{aligned}
$$

With the vertex form of the quadratic we can now easily graph our function $F(x)$.



Let's consider one more example.
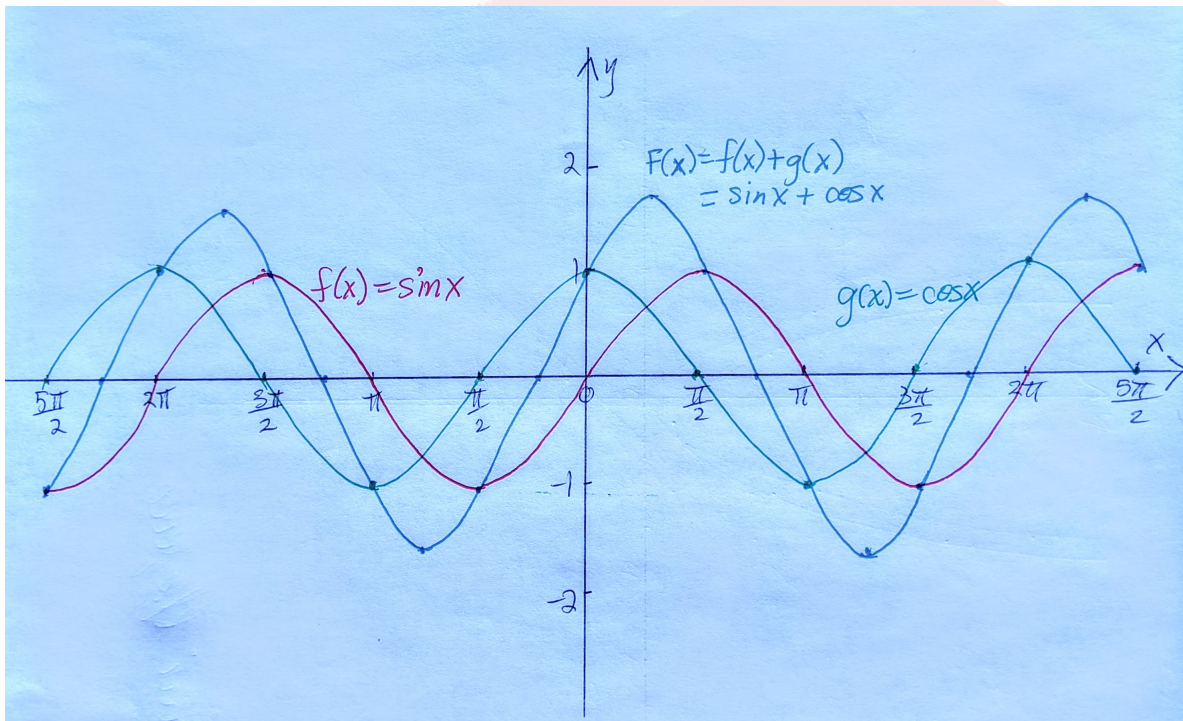
**Example**

Consider the functions,

$$f(x) = \sin x \quad \text{and } g(x) = \cos x$$

Find the sum of $f$ and $g$ and then graph the sum.

**Solution:**

$$F(x) = f(x) + g(x) = \sin x + \cos x$$

It's probably easier to graph $F(x)$ by either considering a table of values or looking at the graphs for $\sin x$ and $\cos x$ on the same axes and then graphically adding them.

## Exercises

Given the functions $f(x) = 3x^2 + 4x - 2$ and $g(x) = -x^3 + 2x^2 + 1$ find the following functions,

(a) $f + g$

(b) $f - g$

(c) $4f + 3g$

(d) $-g + 3$

(e) $-2g + 5f$

(f) $7f$

(g) $f + 6g + 9$

(h) $-f + 2g - 3$

(i) $4g - 3f$

(j) $2f - 2g$