

Composition of Functions

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What is the composition of functions?

The composition of functions means, one function is inserted into the another function where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x , or $x = g(t)$ for $f(x)$. Let's take a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the composition of two polynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \text{ and } g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution The composition of f and g looks like,

$$f \circ g(t) \text{ or } f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t + 1), \text{ where } x = g(t) = t + 1 \\ &= (t + 1)^2 + 3(t + 1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.

Let's try another example.

Example

Consider,

$$f(x) = 3x^3 + 2x \text{ and } g(x) = -2x^2 + 3$$

What is $g \circ f(x)$?

Solution: Then the composition of $g \circ f(x)$ or $g(f(x))$ is given by,

$$\begin{aligned} g(f(x)) &= g(3x^3 + 2) \\ &= -2(3x^3 + 2)^2 + 3 \\ &= -2((3x^3)^2 + 2(2)(3x^3) + 4) + 3 \\ &= -2(9x^6 + 12x^3 + 4) + 3 \\ &= -18x^6 - 24x^3 - 8 + 3 \\ \therefore g(f(x)) &= -18x^6 - 24x^3 - 5 \end{aligned}$$

Exercises

Evaluate $f \circ g(x)$ where f and g are given below.

(a) $f(x) = 3x^2 + 2$ and $g(x) = 4x + 3$

(b) $f(x) = x^2 + 3x + 2$ and $g(x) = 2x^2 + 1$

(c) $f(x) = x^3 + 2x - 4$ and $g(x) = -3x + 1$

(d) $f(x) = x^6 + x^4 + 1$ and $g(x) = x^2 - 2$

(e) $f(x) = x^5 + 3x^2 + x - 1$ and $g(x) = x - 1$