Composition of Functions - Identifying f and g

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What is the composition of functions?

The composition of functions means, one function is inserted into the another funcionn where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x, or $x = g(t)$ for $f(x)$. Let's a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the compostion of two plynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \ \textit{and} \ g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution The compoistion of f and g looks like,

$$f \circ g(t) \ \textit{or} \ f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t+1), \text{ where } x = g(t) = t + 1 \\ &= (t+1)^2 + 3(t+1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.

Evaluating a composite function

Let's try another example but this time we want to evaluate the composite function $f \circ g$ at a particular value for x.

Example

Given $f(x) = 4x + 1$ and $g(x) = -x^2$. Determine, (a) $f(g(-1))$ and (b) $g \circ f(0)$.

Solution:

a) There are two ways we can apprach this exercises.

1. We can first determine what the composite function $f(g(x))$ looks like then let $x = -1$ and evaluate. Let's do this and see what we get.

$$\begin{aligned} f(g(x)) &= f(-x^2) \text{ where } g(x) = -x^2 \\ &- -4x^2 + 1 \end{aligned}$$

Now we let $x = -2$ in our function $f(g(x)) = -4x^2 + 1$.

$$\begin{aligned} f(g(-1)) &= -4(-1)^2 + 1 \\ &= -4 + 1 \\ &= -3 \end{aligned}$$

Therefore, $f(g(-1)) = -3$.

2. The second way to determine the value $f(g(-1))$ is to first find the value $g(-1)$, then plug in this new value in for x in f. Let's do this.
   i. First: $g(-1) = -(-1)^2 = -1$.
   ii. Second: $f(g(-1)) = f(-1) = 4(-1) + 1 = -4 + 1 = -3$

   Therefore, $f(g(-1)) = -3$

So, both ways gave the same answer. The second way, is much quicker.

b) Let's use the second way to find $g \circ f(0)$.

1. First we need to find $f(0)$. $f(0) = 4(0) = 1 = 1$
2. Second, we need to evaluate g at $f(0) = 1$. So, $g(f(0)) = g(1) = -(1)^2 = -1$.

Therefore, $g \circ f(0) = -1$.

Exercises

Given $f(x) = 2 - x^2$ and $g(x) = -3x$, determine the following values,

(a) $f(-1)$

(b) $g \circ f(-1)$

(c) $g(0)$

(d) $f(g(0))$

(e) $f(2)$

(f) $g(f(2))$

(g) $f \circ g(-2))$

(h) $g \circ g(0)$

(i) $f(f(1))$

